

Fixed Price Equilibria on Peer-to-Peer Platforms: Lessons from Time-Based Currencies*

Berno Buechel[†] & Philemon Krähenmann[‡]

December 19, 2018

Abstract

Online, there are many platforms for peer-to-peer exchange, on which participants can trade certain goods or services among each other. Typically, these platforms introduce a platform-specific currency and fix prices to some extent. We model such platforms as pure exchange economies and characterize all fixed price equilibria. We discuss the inherent inefficiency following from the combination of fixed prices and voluntary trade and show that simple additional Pareto improving trades exist. Our theoretical analysis predicts that fixed prices lead on the one hand to less trade, but on the other hand to lower inequality than flexible prices. An empirical investigation of several platforms covering around 100k transactions illustrates that the observed patterns are fully in line with our predictions. This is informative for the market design of peer-to-peer platforms and for markets with price restrictions more generally.

JEL-Code: *D51, D61, D63, L14*

Keywords: *Platforms, P2P, Non-competitive prices, Welfare, Inequality, Walrasian equilibrium*

*We thank Stefan Buehler, Jean-Jacques Herings, Joao Montez, Bodo Hilgers, and Walter Trockel for helpful comments. Special thanks go to Richard Faltings and Bénédicte Droz for excellent research assistance. Berno Buechel gratefully acknowledges research support by the Swiss National Bank (SNB) provided to the Department of Economics of the University of Fribourg.

[†]University of Fribourg, Department of Economics, Bd. de Pérolles 90, CH-1700 Fribourg, Switzerland; Email: berno.buechel@unifr.ch. Web: www.berno.info.

[‡]University of St. Gallen, Institute of Economics (FGN), Varnbuelstrasse 19, 9000 St. Gallen, Switzerland. Email: philemon.kraehenmann@unisg.ch.

1 Introduction

Platforms for peer-to-peer exchange have recently popped up all around the world and for various kinds of goods. Members can trade there as on conventional marketplaces, except that one cannot solely be a buyer or a seller. Instead, every participant must be buyer and seller to some extent. This feature is typically guaranteed by a platform-specific currency that can be earned only through sales on the platform and that can be used only for purchases on it. There are different reasons why a platform operator might want to create such a closed exchange marketplace. First, because it provides incentives for interested buyers to also contribute as a seller. Second, because it commits sellers to spend their earnings among the participants.¹ Third, it is a way of excluding certain sellers, e.g. some platforms exclude professional sellers in order to differentiate themselves from other platforms. Examples for platforms of peer-to-peer exchange are *guestoguest.com*, where members can rent homes with *guestpoints*. These points can only be earned by renting one's own home to other members, while the maximal price one is allowed to charge depends on defined house characteristics. On *bookmooch.com* members can swap goods, where each book costs exactly one point. Further, so-called time banks allow for local exchange of services, where one hour of service is typically fixed to cost one hour of a time currency. As these examples show, many of these platforms restrict price setting. Their motivation to do so could be to guarantee some price stability on the platform, to increase market transparency, or for some kind of fairness considerations.² However, the consequences of the platform operator's decision to keep prices rather fixed or rather flexible are not well understood.

In this paper we model such marketplaces and study the effect of price setting restrictions on efficiency, extent of trade, and equality. We believe this is interesting for at least two reasons. First, peer-to-peer exchange has become common in the Internet. Even though marketplaces where members have to be active on the demand and supply side have existed at least since the nineteenth century (see e.g. Warren, 1852), such systems have become more popular when internet lowered transaction costs. This development is similar to the increased use of online platforms such as eBay, Amazon, and Alibaba to trade more goods from consumer to consumer than it would have been possible with garage sales. Shedding light on the workings of

¹See Mailath, Postlewaite, and Samuelson (2016) for a formalization of that argument.

²We will discuss the reasons to keep prices fixed in some more detail in section 5, i.e. when we can relate to our results.

such platforms is informative for their market design. In particular, rules of price setting, which we can study with our framework, seem to be a crucial feature of a platform’s market design.

Second, economists have been interested in general equilibrium effects in closed exchange economies for a long time. Peer-to-peer exchange platforms are wonderful real-world examples for such closed exchange economies. Hence, we can make use of a rich body of theoretical work, in particular, on the properties of equilibrium allocations with and without Walrasian prices, and link this theoretical work to recent empirical observations.

We model a simple exchange economy with fixed prices. Each agent can offer his endowment and consume goods that are offered by others. Goods can only be traded for a platform-specific currency. To keep the model simple, agents are assumed to have additively separable preferences, which are quasi-linear in the currency and strictly convex.³ We look for fixed price equilibria. The corresponding equilibrium concept is provided by Maskin and Tirole (1984) and refers back to Grandmont (1977), among others. These authors call it K -equilibrium and show that it naturally incorporates the properties of the formerly introduced Drèze equilibrium and Bénassy equilibrium. In particular, a fixed price equilibrium requires that no agent can be forced to trade (“voluntariness”) and that there is no pair of agents who can improve by trading some good (“weak order”). When the fixed prices happen to coincide with Walrasian equilibrium prices, then the fixed price equilibrium and the Walrasian equilibrium allocations coincide. Otherwise, fixed prices necessitate that some agents are constrained from buying or from selling certain goods.

Assuming quasi-linearity of preferences allows us to characterize all fixed price equilibria and to derive empirical predictions about the effect of price setting restrictions in these markets. The starting point of our analysis is the distinction between *scarce* goods and *non-scarce* goods. The former ones are goods for which market demand at given prices is larger than the total endowment. For non-scarce goods market demand is smaller. We show that in any fixed price equilibrium, sellers providing a scarce good keep their optimal amount of that particular good (while all buyers receive at most their desired amount). Further, all buyers receive their optimal amount of each non-scarce good, while the seller of the non-scarce good keeps the rest, which is more than this agent desired. In other words, the seller

³We tailor the assumptions to the application and keep the model simple. This buys us clear-cut results that make the underlying effects transparent. We study robustness to relaxing the assumptions in Appendix B.

of a non-scarce good is constrained from selling the desired amount, and at least one of the buyers of a scarce good is constrained from buying the optimal amount. The rationing scheme therefore only affects the allocation of scarce goods, but not the allocation of non-scarce goods, which must be the same in every fixed price equilibrium.

The first implication of this characterization is that, under very weak conditions any fixed-price equilibrium is not only Pareto *inefficient*, but also *constrained inefficient*. Indeed, we can construct simple chains of bilateral trades that are Pareto improvements within the given price system, under weak conditions on the existence of either strictly scarce or strictly non-scarce goods. Thereby, each bilateral trade either involves agents who are constrained sellers of a non-scarce good and can sell more of their good, or constrained buyers of a scarce good who can buy more of this good. In the simplest case there are two suppliers of non-scarce goods who have a non-zero demand for each other's good. Then they can both improve by exchanging their services. However, in a market with fixed prices this will not occur because both value the numeraire good (currency) more than the consumption of the other's good. In that sense, the price of their goods is "too high." The case with "too low" prices works similarly, and there are also combinations of the two.

We then proceed by comparing fixed price equilibria with Walrasian equilibria. It turns out that the extent of trade in the Walrasian equilibrium is larger than in any fixed price equilibrium. That is, every agent can sell weakly less in a fixed price equilibrium than in the Walrasian equilibrium. In the generic case that a good is strictly scarce or strictly non-scarce, in any fixed price equilibrium the amount traded of any good is even strictly smaller than in the Walrasian equilibrium. Hence, it becomes apparent that fixed prices hamper trade, which is a clear downside of most such platforms. However, Walrasian equilibria do not Pareto dominate fixed price equilibria in general, such that both regimes generate their "winners" and "losers." The winners of flexible prices are typically suppliers of scarce goods because they sell more and at a higher price. As a consequence, inequality is often larger under flexible prices than under fixed prices. We finally investigate data from seven time exchange markets, covering almost 100,000 transactions. These are peer-to-peer exchange platforms, facilitating decentral trade typically through a time-based currency. Prices are fixed to different degrees. We observe that those platforms with fixed prices indeed have lower trade volume and tend to exhibit lower income inequality than those with rather flexible prices. Hence, the empirical patterns are perfectly in line with our model predictions.

Our paper makes three contributions. First, we show that price restrictions, which are a very common feature of peer-to-peer platforms, come at a very high cost. We show theoretically and illustrate empirically that under fixed prices participants leave out many Pareto improving trades, even within the given price regime. The relatively low number of transactions and the correspondingly low trade volume indicate that price restrictions seriously hamper the working of the market.

Second, we show that a potential benefit of price restrictions is that they may lead to more equal market outcomes. Equality of the income distribution is strongly related to the perceived fairness of allocations (e.g. Alesina and Angeletos, 2005, Almås, Cappelen, Sørensen, and Tungodden, 2010, Bénabou and Tirole, 2006). Hence, it may well be that platform operators and market participants who consider the fixed prices of a given platform as more “fair” have a point.

Third, we apply general equilibrium theory, in particular on Walrasian and fixed prices in exchange economies, to a new setting and derive predictions that can be empirically tested. It is well known that non-Walrasian market allocations are generally not Pareto efficient. Moreover, it has been shown that such allocations typically do not even satisfy *constrained efficiency*, that is, there exist Pareto improving trades within the given, non-Walrasian price regime (Younés, 1975; Maskin and Tirole, 1984; Herings and Kononov, 2009). We do not only show for the application of peer-to-peer platforms that this insight applies, but we characterize the inefficiency more specifically by showing how “too high” or “too low” prices prevent some simple Pareto improving trades. In comparison to Herings and Kononov (2009), we make more simplifying assumptions on the utility functions of the market participants, but stay more general in terms of admitting boundary solutions and not imposing a particular rationing scheme. We think that in our application and in many others it is an important feature that a given participant need not buy all products that are in the market and that equal rationing is a very stylized assumption.

We think that our results are also informative for market design outside of peer-to-peer platforms. In many real-world markets prices are (at least in the short run) non-Walrasian. There are several causes of price stickiness, such as costs of changing marketing activities, consumers’ perceptions of clear or “fair” prices, or governmental regulations. Our analysis of closed exchange economies suggests that on many more markets price restrictions hamper trade, induce an inefficiency even in the given price regime, but can contribute to the equality of the market outcomes.

In the next section, we introduce the model. Section 3 presents the results. The empirical illustration follows in section 4. In section 5 we discuss advantages and

disadvantages of fixed prices for peer-to-peer exchange platforms, before we conclude in section 6. All proofs are relegated to the Appendix.

2 Model

Consider a pure exchange economy with $n \geq 2$ agents indexed by i ($i = 1, 2, \dots, n$) and $m + 1$ goods indexed by h ($h = 0, 1, \dots, m$). A price vector $p \in \mathbb{R}^{m+1}$ with $p_0 = 1$ and $p_h > 0$ is exogenously fixed. Each agent i is characterized by a convex consumption set $X^i \subseteq \mathbb{R}^{m+1}$ and an endowment $\omega^i \in X^i$. Each agent i has complete and transitive preferences \succsim^i over consumption bundles X^i , represented by a utility function $U^i : X^i \rightarrow \mathbb{R}_+$. We assume that preferences are continuous and strictly convex.

For the main part of our analysis, we make the simplifying assumption that each agent is endowed with exactly one good such that $\omega_h^i > 0$, while $\omega_h^i = 0$ for $h \neq i$ and $m = n$. This assumption is tailored to the example of house exchange and of service exchange, while the intuition easily extends to the more general case.⁴

For the application of service exchange with a time-based currency, we consider the following interpretation of the model. Each agent j can provide one service $h = j$. A service j is quantified by the amount of time agent j needs to provide that service. Thus, agent i receives one hour of another agent j means that agent j provides an amount of service to agent i , which costs him one hour. Let x_j^i be the amount of time that j stands in the service of i . We denote by $u_j^i(x_j^i)$ the utility agent i derives from service of agent j . Services are priced on that basis. Each hour of service costs one amount of the numeraire good $h = 0$, so $p \equiv (1, \dots, 1)$. The numeraire good is not a service but a time-based currency. For the application of goods that are not services we can immediately interpret x_j^i as the quantity i consumes of the good bought from agent j .

We focus on preferences that are additively separable and quasi-linear in the numeraire.⁵ Utility of agent i is given by:

$$U^i(x^i) = x_0^i + u_1^i(x_1^i) + \dots + u_i^i(x_i^i) + \dots + u_n^i(x_n^i).$$

We assume that u_h^i is twice differentiable with marginal utility $mu_h^i(x_h^i) > 0$ and

⁴Relaxing this assumption is straightforward (see Appendix B.1). The consequences for the results are not severe, but the simple exposition would suffer.

⁵This assumption simplifies the analysis by making demand in one market independent from constraints in other markets. We relax the assumption in section B.2 in the Appendix.

$$\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} < 0 \text{ for all } i, h \text{ and } x_h^i.$$

Let us now turn to the equilibrium concept. As is well-known, for fixed prices we can in general not expect the feature of Walrasian equilibrium that individual optimal decisions are consistent with market clearing. Instead some agents are constrained from selling or from buying on certain markets. The corresponding equilibrium concepts for fixed prices (i.e. in general non-Walrasian prices) are based on two fundamental principles:

- (i) *voluntariness*: no agent can be forced to trade. (Otherwise, his choice could be inconsistent with his preferences.)
- (ii) *weak order*: no two agents can be constrained on two different sides of the same market. (Otherwise, they could improve by trading.)

We precisely follow Maskin and Tirole (1984) by defining a fixed price equilibrium based on these two principles. For this purpose, we need some additional notation. Agent i 's consumption bundle x^i , can be captured by his net trades t^i :

$$x^i = \omega^i + t^i$$

and likewise we can construct the set of possible trades $T^i = \{x^i - \omega^i | x^i \in X^i\}$ of agent i . Since in our context there is only one seller on each market, the endowments are of the form $\omega^i = (0, \dots, 0, 1, 0, \dots, 0)$. For all $i \neq h$ we therefore have $x_h^i = t_h^i$ with weakly positive t_h^i ; and for $i = h$ we have $x_h^i = \omega_h^i + t_h^i$ with weakly negative t_h^i . Let $\tilde{T}^i := T^i \cap \{t^i | p \cdot t^i = 0\}$ be the set of (with respect to budget) feasible net trades of agent i . $\tilde{T} = \{(t^1, \dots, t^n) \in (\tilde{T}^1, \dots, \tilde{T}^n) | \sum_i t^i = 0\}$ is then the set of feasible net trades in the economy. We define $\tilde{\tau}_h^i(t^i) := \{\tilde{t}^i \in \tilde{T}^i | \tilde{t}_k^i = t_k^i \ \forall k \neq 0, h\}$, as the (budget) feasible net trades of agent i that coincide with the net trades t^i on all markets, but on market h and 0. Finally, let $Z = ((\underline{Z}^1, \bar{Z}^1), \dots, (\underline{Z}^n, \bar{Z}^n))$ be a vector of quantity constraints such that $\underline{Z}^i \leq 0$ and $\bar{Z}^i \geq 0$ and $\underline{Z}_0^i = -\infty$ and $\bar{Z}_0^i = \infty$ for all i .

We can now define equilibrium allocations x under fixed prices p by defining the corresponding equilibrium trades t .

Definition 1 (Fixed Price Equilibrium, Maskin and Tirole, 1984). *A fixed price equilibrium (FPE) is a vector of (fully) feasible net trades $t \in \tilde{T}$ associated with a vector of quantity constraints Z such that for all i ,*

- (V) *exchange is "voluntary:" t^i is the \succsim^i -maximal element among the (budget) feasible net trades $\tilde{t}^i \in \tilde{T}^i$ that satisfy the constraints $\underline{Z}^i \leq \tilde{t}^i \leq \bar{Z}^i$.*

(WO) exchange is “weakly orderly:” if for some commodity h , and some agents i, j , there is a trade $(\tilde{t}^i, \tilde{t}^j) \in \tau_h^i(t^i) \times \tau_h^j(t^j)$ such that $\tilde{t}^i \succ^i t^i$ and $\tilde{t}^j \succ_j t^j$, then $(\tilde{t}_h^i - \bar{Z}_h^i)(\tilde{t}_h^j - \underline{Z}_h^j) \geq 0$. In words: if there is a feasible trade that only differs from trade t on market h and on market 0 and both traders i and j would benefit from that trade, then it cannot be that the two traders are at different sides of the market in the sense of one wanting to buy less (respectively to sell more) and the other wanting to buy more (respectively to sell less).

Voluntariness (V) captures that individual agents optimize across all markets, given their constraints Z^i . $\underline{Z}^i \leq 0$ ($\bar{Z}^i \geq 0$) then ensures that i cannot be forced to buy (sell). Weak order (WO) captures that there is no pair of agents i, j who can both strictly improve by making an (additional) trade on a single market h , when the constraints on this market are relaxed. Such a trade can either be between a seller and a buyer who exchange good h for money; or between two buyers who change the amount they buy of good h without changing the total demand (for seller h).

Weak order (WO) is equivalent to the following property, which is actually used in Maskin and Tirole (1984): there is no market $h(\neq 0)$ in which a Pareto improvement can be reached when ignoring the constraints on this market and keeping all other markets (except the market for the numeraire 0) fixed.⁶

3 Results

An agent i can only afford consumption bundles x^i that are in his budget set $\tilde{X}^i(p) = \{x^i | p \cdot x^i \leq p \cdot \omega^i = p_i \omega_i^i\}$. For fixed prices p , compute demand \hat{x}^i of an agent i as $\hat{x}^i := \operatorname{argmax}_{x^i \in \tilde{X}^i(p)} U^i(x^i)$, i.e. the consumption bundle that maximizes agent i 's utility within the budget set.

Definition 2 (scarce and non-scarce goods). *Good h is called scarce if there is no excess supply (at fixed prices p), i.e. if $\sum_{i \in N} \hat{x}_h^i \geq \sum_{i \in N} \omega_h^i = \omega_h^h$. Otherwise, it is called non-scarce.*

Scarce goods are in high demand, relative to their supply, while non-scarce goods are not. The following lemma shows that scarcity of a good h can be inferred by comparing the given fixed price p_h with the Walrasian equilibrium price p_h^* .⁷

⁶This notion is called “weak order (O)” in Maskin and Tirole (1984). We show the equivalence of the two notions in appendix A.1.

⁷Due to our assumptions on preferences, the Walrasian equilibrium is unique.

Lemma 1. *Let p^* denote the price vector of the Walrasian market equilibrium. Good h is scarce (at fixed prices p) if and only if $p_h^* \geq p_h$.*

3.1 Characterization of fixed price equilibria

Proposition 1 (Characterization). *In every FPE, each good $h \neq 0$ is allocated as follows:*

- (a) *If h is non-scarce, every buyer receives the desired amount, while the seller keeps the rest. That is: $\forall i \neq h, x_h^i = \hat{x}_h^i$ and $x_h^h = \omega_h^h - \sum_{i \neq h} \hat{x}_h^i (> \hat{x}_h^h)$.*
- (b) *If h is scarce, every buyer receives at most his desired amount, while the seller keeps (exactly) the desired amount. That is: $\forall i \neq h, x_h^i \leq \hat{x}_h^i$ and $x_h^h = \hat{x}_h^h$.*

Proposition 1 provides a clear-cut characterization of all FPE. It fully determines the allocation of all non-scarce goods and it determines the allocation of all scarce goods up to a rationing scheme. In the literature equal rationing is sometimes imposed (e.g. Herings and Konovalov, 2009). Our results hold for all FPE and hence for all rationing schemes. Note also that Proposition 1 holds without any assumption on \hat{x}_h^i being interior. In particular, $\hat{x}_h^i \in \{0, \omega_h^h\}$ is admitted and does not change the statement. Such a clear characterization of all FPE is due to our assumptions on the utility function. Demand in one market is not affected by quantity constraints in another market. The rationing scheme for good h therefore only affects demand of good h and the numeraire. The asymmetry in the strength of the two statements (a) and (b) follows from the assumption that every agent is only endowed with one good, which means that every agent can only sell one good, while he can buy any good.⁸

If a good h is scarce but not strictly scarce, then the inequality of the second statement of the Proposition 1 holds in fact with equality. Since Walrasian prices p^* have the feature that each good h is scarce, but not strictly scarce, it follows that in Walrasian equilibrium, which is a special case of a FPE, no agent is constrained, while markets clear. However, for generic prices $p_h \neq p_h^*$ at least one agent is constrained from buying the desired amount of a scarce good h and the seller of a non-scarce good h is constrained from selling the desired amount.

⁸Relaxing this assumption, would lead to results for non-scarce goods that are fully analogous to the results with scarce goods (see Appendix B.1). Such results are slightly weaker since the allocation of non-scarce goods then also depends on the rationing scheme. However, loosening this assumption would not undermine the substance of the results.

3.2 Inefficiency of Fixed Price Equilibria

We now turn to efficiency.

Definition 3 (Pareto Efficiency and Constrained Efficiency). *An allocation x is Pareto efficient (PE) if $\nexists x' = x + t$ with $\sum_i t^i = 0$ which Pareto dominates x . An allocation x is constrained efficient (cPE) if $\nexists x' = x + t$ with $t \in \tilde{T}$ which Pareto dominates x .*

The notion of Pareto efficiency is stronger than the notion of constrained efficiency because it admits more general improvements. For Pareto efficiency we consider any other allocation that is feasible, while constrained efficiency only considers allocations that obey the budget feasibility for the fixed prices p . Instead of requiring that every agent's wants to consume a strictly positive amount of every good, i.e. interiority, we make a much weaker assumption on the attractiveness of different goods.

Definition 4 (Weak Interiority). *An economy satisfies weak interiority if the following holds for every market h .*

- (i) *If h is non-scarce, then there is another non-scarce good $k \neq h$ such that $\hat{x}_k^h > 0$, i.e. the seller of a non-scarce good h demands at least one other non-scarce good.*
- (ii) *If h is scarce, then $\hat{x}_h^h > 0$, i.e. the seller of a scarce good h demands a positive amount of it.*

With these notions in hand, we can formalize the inefficiency, not only with respect to Pareto efficiency, but also with respect to constrained efficiency.

Proposition 2 (Inefficiency). *Suppose a non-scarce good h and at least one agent $i \neq h$ exist such that $\hat{x}_h^i > 0$. Then no FPE is Pareto efficient. Suppose $p_h^* \neq p_h$, $\forall h$ and weak interiority is satisfied. Then no FPE is constrained efficient.*

The first statement of Proposition 2 is a standard inefficiency result. In the proof of the second part, we show that under the condition of weak interiority, there is a chain of agents such that each pair in the chain can strictly improve by bilateral trade on a single market.⁹ The inherent type of inefficiency emerging from the combination of fixed prices and decentralized trade is easiest to see by assuming

⁹In the terminology of Herings and Konrad (2009), this means that no fixed price equilibrium is “B-p efficient”, which is an even weaker notion of efficiency than constrained efficiency.

prices fixed to $p \equiv (1, \dots, 1)$ and interiority. By Proposition 1 a supplier i of a non-scarce good derives then a marginal utility of 1 from each non-scarce good $h \neq i$. However, his marginal utility from good i is strictly smaller. Therefore, any two suppliers of a non-scarce good could improve by exchanging some amount of their goods directly, without using the numeraire good in the transactions. This will not occur because both value the numeraire good (currency) more than the consumption of the other's good. In some sense the prices of the two goods are "too high." A similar issue occurs for scarce goods: prices are "too low" such that despite the high demand, a supplier of the scarce good is not willing to offer a sufficient amount of it, while she would do so in exchange for another good that she values highly. This shows how decentralized trade fails to enable even simple Pareto improving trades when prices are fixed.

3.3 Fixed price vs. Walrasian equilibrium

We now compare the Walrasian equilibrium and FPE, first with respect to the amount traded and then with respect to inequality.

Proposition 3 (less trade). *In every FPE t , the total amount traded of any good $h \neq 0$ is smaller than in the Walrasian equilibrium t^* , i.e. $\sum_{i \neq h} t_h^i \leq \sum_{i \neq h} t_h^{i,*}$. For non-scarce goods h , every single buyer $i \neq h$ buys less than in the Walrasian equilibrium, i.e. $t_h^i \leq t_h^{i,*}$, $\forall i \neq h$.*

This result follows from Proposition 1 and the fact that the demand of each good is decreasing in its own price. For the interpretation, suppose that the fixed price p_h of a good h does not coincide with the Walrasian price p_h^* . If h is non-scarce, $p_h^* < p_h$ (Lemma 1). Since buyers of non-scarce goods are not constrained (neither in the FPE nor in the Walrasian equilibrium), they would buy more in the Walrasian equilibrium. If good h is scarce, $p_h^* > p_h$ (Lemma 1). Since sellers of scarce goods are not constrained (neither in the FPE nor in the the Walrasian equilibrium), they would sell more in the Walrasian equilibrium.

The result on less trade also has implications for inequality of incomes. Let $y = (y^1, \dots, y^n)$ denote the income distribution with $y^i = |t_i^i| \cdot p_i$. Since suppliers of scarce goods sell less with fixed prices (by Proposition 3) and fixed prices are lower than flexible prices in equilibrium (by Lemma 1), their income is lower under fixed prices, i.e. $y^i = |t_i^i| \cdot p_i < |t_i^{i,*}| \cdot p_i^* = y^{i,*}$. Suppliers of non-scarce goods also sell less in the FPE, but fixed prices for their goods are higher than Walrasian prices. Whether

the overall effect on income is positive or negative depends on the price elasticity of demand. The relevant prices are $p = (1, p_1, \dots, p_n)$ and $p^* = (1, p_1^*, \dots, p_n^*)$ and the corresponding demand is $Q_h := \sum_{i \neq h} \hat{x}_h^i$ and $Q_h^* := \sum_{i \neq j} x_h^{i,*}$. Hence, we define the (discrete) price elasticity of demand as $\varepsilon_h := \frac{\Delta Q_h}{\Delta p_h} \cdot \frac{p_h}{Q} = \frac{Q_h^* - Q_h}{p_h^* - p_h} \cdot \frac{p_h}{Q_h}$. With two strong conditions that we define next, we can compare FPE with Walrasian equilibria in terms of inequality.

Assumption 1. *We assume that there is at least one scarce good and one non-scarce good and define two qualifications.*

- (i) *Suppose at prices p the supply for every scarce good i is larger than the demand for every non-scarce good j weighted by the prices, i.e. $S_i = \omega_i^i - \hat{x}_i > Q_j \cdot \frac{p_j}{p_i}$.*
- (ii) *Suppose demand for every non-scarce good j is inelastic or isoelastic, i.e. $|\varepsilon_j| \leq 1$.*

Corollary 1 (Inequality). *Suppose Assumption 1 holds and suppose that every good h faces positive demand for the fixed price p , i.e. $Q_h > 0$. Then moving from any fixed price equilibrium to the Walrasian equilibrium increases inequality in the following sense: Those with the highest income increase their income, while the income of all others does not increase.*

The result is based on our distinction of scarce and non-scarce goods. Under Assumption 1 (i) suppliers of scarce goods earn more than suppliers of non-scarce goods already under fixed prices. Hence, those with the highest income are the suppliers of scarce goods. When moving to flexible prices, their income increases because both the quantities sold and the prices increase. For suppliers of non-scarce goods on the other hand, Assumption 1 (ii) implies that their income does not increase when moving from fixed to flexible prices because the reduction in prices cannot be compensated by the increase of sold goods. This is due to the inelastic demand. Hence, Corollary 1 can also be phrased as “the rich get richer and the poor get poorer” where the “rich” are the suppliers of scarce good and the “poor” the suppliers of non-scarce goods.

This is a genuine increase of inequality. It also links to several common measures and indices of inequality. First, the share of income of the, say, top 25% increases when 25% is the fraction of suppliers of scarce goods. Another simple and common measure takes the ratio of two incomes, comparing a certain percentile, e.g. 10%, with another percentile, e.g. the median. Also this measure of inequality increases when the percentiles are taken such that they compare suppliers of scarce goods with

suppliers of non-scarce goods. Several inequality indices are decomposable in a well defined way into inequality within groups and inequality between groups (e.g. Cowell, 2000). In particular, this is true for the Theil index (Foster, 1983). Defining groups by suppliers of scarce and non-scarce goods, we get that the inequality between groups increases when moving from FPE to Walrasian equilibrium. However, there is no clear implication for the inequality within groups such that we cannot exclude that inequality within groups falls extremely and dominates the rise of inequality between groups. Similarly for the Gini coefficient.¹⁰ The Gini coefficient is usually defined as the area between the Lorenz curve and the id line. When, however, formalized as a normalized sum of absolute differences, we can see that all differences between groups, say i is supplier of a scarce good and j of a non-scarce good, $|y^i - y^j| < |y^{i,*} - y^{j,*}|$, unambiguously increase.

While Corollary 1 has strong implications, it is notably based on a very demanding assumption: Assumption 1. In reality, we would expect that both parts of Assumption 1 are not fully satisfied. (i) There will not be a perfect separation between suppliers of scarce and non-scarce goods in fixed price equilibrium with all suppliers of scarce goods at the top of the income distribution. (ii) There will be suppliers of non-scarce goods who benefit from flexible prices because their reduction of selling price is over-compensated by the increase in the amount sold. However, the main force that drives the inequality result of Corollary 1, will still be at work. Suppliers of scarce goods heavily benefit from the introduction of flexible prices. The boost of their income is due to the combination of larger amounts sold and higher prices, while suppliers of non-scarce goods face lower prices. We consider it as likely that this boost of income increases inequality even if the qualifications of Assumption 1 are not met.

Comparisons of income distributions have to be distinguished from welfare comparisons. Whether an agent is better off in the Walrasian equilibrium or in the FPE depends not only on her income, but also on the prices of the goods she demands, and on the quantity constraints she faces at the scarce goods. In general, the Walrasian equilibrium does not Pareto dominate a given FPE; and neither the other way around.

¹⁰For the Gini coefficient this decomposability does not hold in general (there is also an interaction term), but it holds when the groups are non-overlapping (Cowell, 2000), which is indeed true under our Assumption 1 because all suppliers of scarce goods are earning more than all suppliers of non-scarce goods.

4 An Empirical Illustration

4.1 The Data Set

Our theoretical investigation provides clear-cut results on how goods are allocated when prices are fixed and how the allocation differs from Walrasian equilibrium. The model applies in particular to time exchange markets. These are the purest real-world examples of exchange economies we can think of. Concretely, these are marketplaces for service exchange, which facilitate decentral trade through a time-based currency. Often, but not always, all prices are fixed and equal, e.g. any hour of service yields one hour on the time account for the supplier and costs one hour for the consumer. Such markets have existed at least since the nineteenth century (see e.g. Warren, 1852), but it was much more recently that many such markets have popped up all around the world.¹¹ We now set out to describe real transaction patterns of several such platforms in order to check whether these patterns are consistent with our model predictions.

For seven platforms, we obtained data of all transactions made between 2008 and 2016.¹² These 100,000 odd transactions were all managed by the same software and are hence directly comparable. For each platform the recordings of the transactions begin with the introduction of the software. Each platform has a set of rules on how to trade on them. These rules are highly similar to each other on all platforms with one main difference: Prices are fixed to a higher or lower degree. One platform writes [translated from German]: “The exchange rate for performance is 1:1 – one hour of performance entitles to receive one hour of counterperformance.” Three other platforms have similar formulations to fix prices.¹³ At the other end of the spectrum, there are platforms that only suggest a certain price, but leave the choice to the market participants. One of these platforms writes [translated from German]: “We recommend to charge 100 [currency units] per hour. However, the two exchange

¹¹For instance, already in 2011, 300 registered “time banks” have been counted only in the US, which is just one of 34 countries with such institutions (Cahn, 2011). There is a broad range of services offered, from ironing clothes, mowing someone’s lawn to looking after children, or teaching a certain craft.

¹²We asked 18 platforms in Austria and Switzerland for their consent to analyze their anonymous transaction data and received a response of 55%, among whom the response was positive in 80% of the cases. One case with positive response was not considered because this data set did not even span one year. When obtaining the data, we agreed not to reveal the identity of these platforms.

¹³Moreover, it is explicitly forbidden to combine transactions with transfers in real currencies, except for costs of material, for which the price of purchase is to be used.

partners agree on the price by themselves.” Other potentially relevant differences concern restrictions of the budget from below or above; and rules on how much to pay each year as a membership fee, and whether companies are admitted.¹⁴ Some platforms explicitly emphasize exchange of services, while on all platforms both services and goods are admitted. In sum, it is however remarkable how similar the rules on these platforms are.

Table 1 provides some summary statistics about the platforms. According to the formulations on how to set prices, and consistent with sample transactions, we organize the platforms into four with fixed prices labeled F1,...,F4 and three with rather flexible prices labeled W1,...,W3. Within both categories the platforms are ordered and labeled according to the length of our recordings (see column *years*). Members are defined as participants who engaged in at least one transaction with another participant. We only consider transactions that take place between participants, not system transactions such as the payment of an annual membership fee. In total, we have data on 2,911 members and of 98,527 transactions.

Among our theoretical results, Proposition 3 and Corollary 1 can be directly taken to the data. They predict that the platforms with rather flexible prices will have more trade and higher inequality. We analyze these two properties in turn.

4.2 Amount of Trade

We assess the amount of trade by two complementary measures. The first measure is the number of transactions. The second measure is the trade volume, i.e. the money in the time-based currency spent on trades (converted to hours in the case of W2 and W3).¹⁵ Both measures are normalized by computing the amount per member per year to make the platforms comparable.¹⁶ The amount traded for both measures is illustrated in Figure 1. The platforms are still ordered by observed years, but organized into the two categories fixed prices (F1-F4) and flexible prices

¹⁴As a test of robustness, we excluded all members that are identifiable as firms. This does not change any of the qualitative results. (In terms of absolute numbers, the trade volume and the inequality on the largest platform, later labelled W3, become more moderate.)

¹⁵We do not have the quantities of many transactions, but we always have the price paid.

¹⁶More precisely, we compute for each member of a platform how much he traded on average per year for all the years that he was active, i.e. had at least one transaction, and average this number over all members. In this way we can account for the fact that individuals can join and leave a platform within the observed years. Another normalization of simply dividing the amount by the age of a platform and the number of members leads to an underestimation of the trade per member per year, but leads to the same qualitative differences between the platforms.

ID	years	members	TA	currency	price recommendation
F1	5.6	215	1,559	hours	Performance is exchanged 1:1 – one hour of performance entitles to one hour of counter-performance
F2	7.9	330	5,094	hours	An exchange rate of 1:1 is assumed. One hour of performance entitles to obtain one hour of performance for personal use.
F3	8.7	324	4,175	hours	The exchange among those willing to trade is accounted in hours and minutes.
F4	9.6	708	12,513	hours	Concerning the exchange of performance the following holds: Each hour has the same value.
W1	5.6	179	2,804	hour-units	Goods and services are generally traded according to time-units.
W2	6.7	118	2,975	units	The exchange partners determine the performance’s value in currency units. As a point of reference, we recommend to value one hour working time by 100 currency units.
W3	10.4	1,037	69,346	units	We recommend to charge 100 [currency units] per hour. But two exchange partners decide on the price themselves.

Table 1: Description of different platforms with time-based currencies. Members are all participants of a platform who had at least one transaction with another member. TA is the total number of transactions on the platform (excluding system transaction). The price recommendation is a literal translation from German. We categorized platforms into those with fixed prices, labeled “F”, and into those with rather flexible prices, labeled “W” for Walrasian, according to the price recommendation and the flexibility of the prices in sample transactions. Platforms ordered first by fixed versus flexible prices and then by time span of data recordings.

(W1-W3) for a better comparison.

The figure clearly suggests that fixed prices (F1-F4) are associated with less trade than the rather flexible prices (W1-W3), as predicted by our model. On average the platforms with fixed prices only trade 12.6 hours per member and year, while those with flexible prices trade 43.9. On average the platforms with fixed prices only have 6.0 transactions per member and year, while those with flexible prices have 16.9. Concerning the trade volume, platform W2 is an exception to the general pattern since its trade volume is in the range of the platforms with fixed prices, but concerning the number of transactions it is consistent with the pattern.

4.3 Inequality

We investigate inequality of income. Each trader’s annual income is the trade volume that he sells in a given year. Inequality typically increases with the length of the

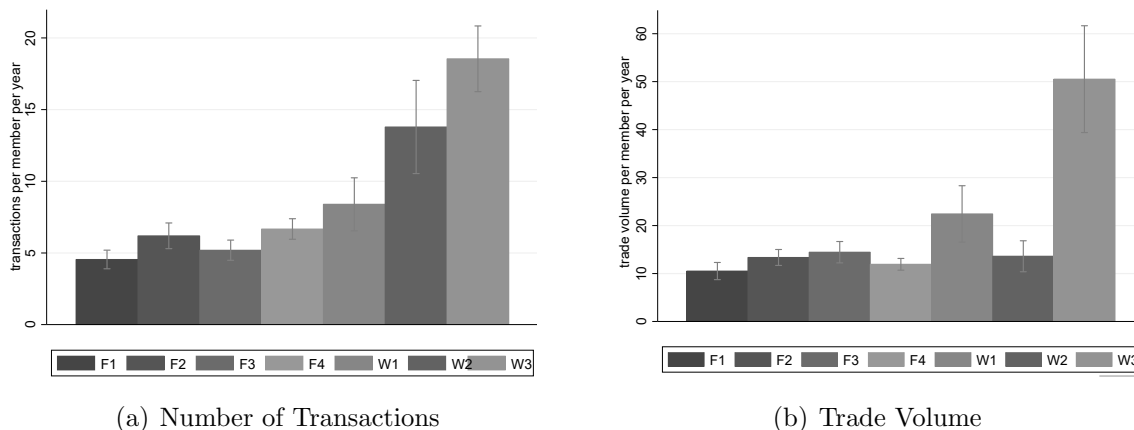


Figure 1: Amount of trade: Panel (a) shows the average number of transactions per member per year. Panel (b) shows the average trade volume per member per year. Confidence intervals are standard 95% confidence intervals based on the heterogeneity between the members.

observed period because some members are active on the platform for a longer time period than others. We therefore compute inequality measures for each fully observed year separately.

We first describe inequality by the ratio of incomes of different percentiles. The inequality result, Corollary 1, has a direct implication for this measure: Given that we compare the income of suppliers of scarce goods with those of non-scarce goods, inequality is larger under flexible prices. Table 2 reports the income of different percentiles in relation to the income of the median percentile. The 95 percentile, that is a top 5% earner, earns 8.32 times the earnings of the median in platform F1 and even 10.04 times the median in platform W1. Considering the 95 percentile and the 90 percentile, inequality is larger in platforms W1 and W3 with flexible prices than on the other platforms. The platforms with fixed prices F1-F4 and platform W2 are similar in terms of inequality.

Interestingly, it is only the relative income of the top earners which is higher in W1 and W2. The relative income of lower percentiles is comparable on all platforms. That is in line with Corollary 1 if the top 10% earners provide a scarce good, while a fraction of the top 25% earners already provide a non-scarce good. Top earners under flexible prices are therefore likely those who provide the “most” scarce goods.

On all platforms relative earnings from the bottom 25% are low. Overall inequality is therefore large. The Gini coefficient is between 63.1 and 68.6. It is again higher for W1 and W3, that is however again driven by the top incomes. Figure C.1 in the appendix shows the Lorenz curve for each platform for each year. The id

id	95q/50q	90q/50q	75q/50q	25q/50q	Gini
F1	8.32	5.62	3.29	0.27	63.1
F2	7.63	5.50	2.59	0.12	63.5
F3	7.78	5.53	2.56	0.20	64.5
F4	8.76	5.89	2.86	0.27	64.6
W1	10.04	6.52	2.62	0.23	67.2
W2	7.33	5.05	2.83	0.21	60.1
W3	11.44	7.84	3.25	0.19	74.1

Table 2: Several measures of inequality. Ratio of income quantiles over median (50q); and Gini coefficient average over all fully covered years.

line is the benchmark of full equality. The other black lines illustrate inequality of platforms with flexible prices W1-W3 and the red gray lines of those with fixed prices. Oftentimes, two black lines – corresponding to W1 and W3 – lie fully below all gray lines, which is known as Lorenz domination. When one distribution Lorenz dominates another one, then the first is more unequal with respect to most inequality measures. Hence, the platforms with flexible prices, apart from W2, lead to greater inequality.¹⁷

A second look at inequality is possible when analyzing the trade networks that emerged on each platform.

4.4 Trade Networks

We analyze the trade networks that are implied by the transactions on each platform. Each member is a node in the network. An arc from trader i to some trader j indicates that i bought a good from trader j . The network hence illustrates the flow of money. Two such trade networks are visualized by Figure 2.

Table 3 reports several network statistics for each platform.¹⁸ The platforms are ordered as before. The number of arcs per node is the average number of business partners a member of the platform has. The density is the fraction of present arcs over all possible arcs. The table suggests that more flexible prices are associated

¹⁷To check whether the differences in inequality are really due to the top earners, we redrew the Lorenz curves for truncated distributions where on every platform the top 10% earners are excluded. Indeed, Lorenz domination is lost by this manipulation.

¹⁸The network statistics are computed by the package *nwcommands* used in the software *STATA 14*.

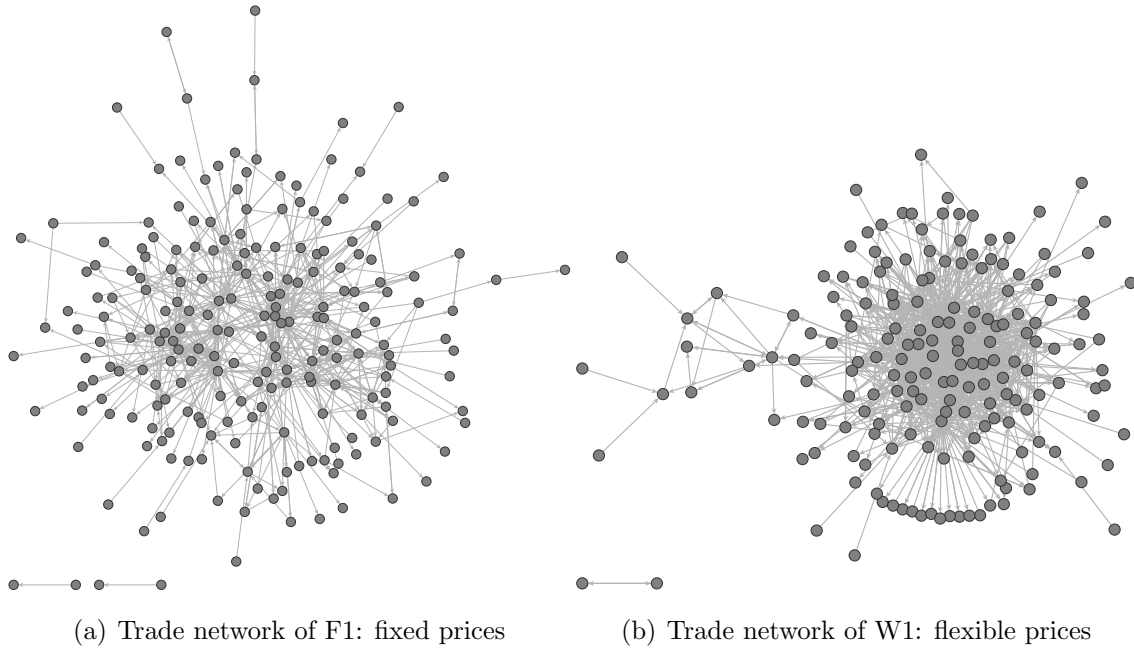


Figure 2: Trade network of platform F1 (panel (a)) and of platform W1 (panel (b)). Both networks are of similar age and of similar size (in terms of number of nodes), but the trade network of platform W1, the one with rather flexible prices, is much denser than the trade network of platform F1.

with a higher density and more arcs per node. On average the platforms with fixed prices only have 5.1 arcs per node (i.e. business partner per member), while those with flexible prices have more than 12.9. This confirms the pattern of more trade for flexible prices and is in line with our theoretical prediction.

Concerning inequality, centralization measures inequality with respect to the number of customers (*indegree*) and to the number of suppliers (*outdegree*). Table 3 shows that centralization is substantially higher for flexible prices than for fixed prices, confirming our result on inequality of income. This is reassuring because differences in the inequality of the yearly income, as reported in Table 2, were more moderate.

id	nodes	arcs	arcs/node	density	indegree centralization	outdegree centralization
F1	215	695	3.2	0.015	0.11	0.09
F2	330	1,593	4.8	0.015	0.13	0.12
F3	324	1,756	5.4	0.017	0.10	0.17
F4	708	4,828	6.8	0.010	0.14	0.11
W1	179	1,197	6.7	0.038	0.27	0.33
W2	118	1,150	9.7	0.083	0.48	0.36
W3	1,037	23,071	22.2	0.021	0.41	0.32

Table 3: Network statistics. Nodes are the members of a platform. Arcs are the trade relationships. Density is the number of present arcs over all potential arcs. Centralization measures inequality with respect to the number of customers (indegree) and of the number of suppliers (outdegree).

5 Discussion

Given the theoretical and empirical findings above, what are the advantages of platforms for peer-to-peer exchange in comparison to other market forms?

The fact that Walrasian equilibria are Pareto efficient, while fixed price equilibria are not, does not mean that the Walrasian equilibrium Pareto dominates the fixed price equilibrium, as noted before. Pareto efficiency of the Walrasian equilibrium however implies that there is always at least one agent who prefers flexible prices. If that agent would leave the platform with fixed prices, then in the new fixed price equilibrium at least one other agent would prefer flexible prices; and so on. At the end of this hypothetical procedure, only one agent would remain.

It is therefore natural to ask, why such platforms with fixed prices can survive among rational agents. One possibility is that agents would like to commit themselves to buy inside the network. Then the platform-specific currency serves as a local currency in the sense of Mailath, Postlewaite, and Samuelson (2016), for which some price stability is considered as necessary. Another reason is transparency. Certain prices are simple and seem focal. If there are high transaction costs for finding mutual agreements on how much to pay for certain services, it can be cheaper to rely on focal prices, which are suggested by a platform operator.

Another possibility is that some participants of these platforms have social preferences. Some prices could be perceived as *fair* such that (a) *procedural fairness* is a motive to engage in these transactions;¹⁹ or it could be that the resulting allocation

¹⁹In fact, the origin of time-dependent currencies is the postulate that every hour of work should

is considered more fair, than the Walrasian equilibrium allocation such that (b) *distributional fairness* is the motive. If the former motive, (a) procedural fairness, is predominant, the question arises whether there are Pareto superior allocations given the restriction that services are only exchanged according to the fixed prices. Our paper provides an answer to this question by showing that the FPE allocations are constrained inefficient and that Pareto improvements often only necessitate simple trades. Participants motivated by procedural fairness could hence either insist on the decentralized trade and accept the corresponding efficiency loss; or agree to a different allocation mechanism, e.g. a matching algorithm, that keeps the same prices, but leads to Pareto superior outcomes. Concerning (b) distributional fairness, our paper shows that fixed prices tend to induce lower inequality of income. If this motive is predominant, the question arises whether there are alternative (market) mechanisms that lead to Pareto superior outcomes, given the agents' social preferences. For instance, more trade without much higher inequality could be induced by a competitive market combined with some redistribution of income (e.g. Alesina and Angeletos, 2005; Bénabou and Tirole, 2006).

6 Conclusion

We have analyzed platforms for peer-to-peer exchange. These are closed exchange economies, on which price setting is often restricted and markets therefore do not clear. Assuming quasi-linear preferences allowed us to characterize the set of fixed price equilibria. Allocations are typically constrained inefficient, i.e. there are Pareto improvements even within the given price system. Moreover, we can show that trade volume under fixed prices is always lower than under competitive prices. Finally, under more restrictive assumptions, inequality must be lower as well. These findings are corroborated by an empirical illustration of several real platforms with time-based currencies.

Our methodological approach is innovative in that it combines traditional economic theory with a current online phenomenon and also makes use of techniques from network analysis. The main results show that fixed prices come at a high cost (since they lead to a constrained inefficient outcome and to less trade than competitive prices). This finding relates back to known inefficiency results (Younés, 1975; Maskin and Tirole, 1984; Herings and Konovalov, 2009), which seem to become vital and tangible in our setting. By investigating and illustrating how platforms for

have the same value (Warren, 1852).

peer-to-peer exchange are affected by fixed prices, we hope to provide lessons that are not restricted to these markets, but can be addressed in many markets with price restrictions.

A Mathematical Appendix

A.1 Lemma A.1

Lemma A.1 (Weak Order). *Property weak order (WO) as defined in Definition 1 is equivalent to the following property of Maskin and Tirole (1984):*

(O^{''}) *exchange is weakly orderly: for all markets h , there exists no alternative (fully) feasible vector $\tilde{t} \in \prod_i \tau_h^i(t^i)$ such that, for each i , $\tilde{t}^i \succsim^i t^i$ with at least one strict preference.*

Proof. Clearly, (O^{''}) implies (WO) because if (WO) is violated, then there exists a pair i, j and a trade $(\tilde{t}^i, \tilde{t}^j)$ which is a Pareto improvement. On the other hand suppose (WO) is satisfied. Then there is no such pair as shown below.

Suppose there is a Pareto improvement \tilde{t} concerning market h . Then at least one agent i must be better off: $\tilde{t}^i \succ^i t^i$. Hence, $\tilde{t}_h^i \neq t_h^i$. Assume first that $\tilde{t}_h^i > t_h^i$ (i.e. i would like to buy more of h or sell less of it). By $\sum_i \tilde{t}_h^i = 0$ there must be some $j \neq i$ with $\tilde{t}_h^j < t_h^j$, i.e. who sells more or buys less of h . Since \tilde{t} is a Pareto improvement $\tilde{t}^j \succsim^j t^j$. Thus, either $\tilde{t}^j \succ^j t^j$ and we are done or $\tilde{t}^j \sim^j t^j$. In the latter case, consider $\hat{t} := \frac{t + \tilde{t}}{2}$. Strict convexity implies that $\hat{t}^j \succ^j t^j$. Moreover, $\hat{t}^i \succ^i t^i$. Now, analogously for $\tilde{t}_h^i < t_h^i$ there is a j with $\tilde{t}_h^j > t_h^j$ and $\tilde{t}^j \succsim^j t^j$. Again, we have either $\tilde{t}^j \succ^j t^j$ or $\hat{t} := \frac{t + \tilde{t}}{2}$ has the required properties. \square

A.2 Proof of Lemma 1

In the Walrasian equilibrium x^* for all agents i consuming a positive amount of good h we have $mu_h^i(x_h^{i,*}) = p_h^*$. Now, suppose $p_h^* \geq p_h$. Then $mu_h^i(x_h^{i,*}) \geq p_h$ for every i consuming a positive amount of good h at the price p^* . Since $mu_h^i(\hat{x}_h^i) = p_h$, $x_h^{i,*} \leq \hat{x}_h^i$ by concavity of u_h^i . Moreover, all agents consuming a positive amount of h at price p^* will do so at price $p_h \leq p_h^*$. Thus, $\sum_{i \in N} \hat{x}_h^i \geq \sum_{i \in N} x_h^{i,*} = \omega_h^h$, where the last equality holds because in the Walrasian equilibrium markets clear. Now, suppose $p_h^* < p_h$, then, for the analogous reasons as above, $\sum_{i \in N} \hat{x}_h^i < \sum_{i \in N} x_h^{i,*} = \omega_h^h$. \square

A.3 Proof of Proposition 1

We prove both statements separately.

(a) Non-scarce good h : Consider an allocation \tilde{x} that does not satisfy this property. Hence, there is a buyer i and a good $h \neq i$ such that $\tilde{x}_h^i \neq \hat{x}_h^i$.

Suppose first $\tilde{x}_h^i > \hat{x}_h^i$, i.e. i receives more than desired. Then $\underline{Z}_h^i \leq 0 \leq \hat{x}_h^i < \tilde{x}_h^i \leq \bar{Z}_h^i$ (for the canonical constraints, the first and the last inequalities are

equalities). Hence, within the constraints and within i 's budget set, i could also reduce the amount that he buys from good h to $\tilde{x}_h^i - \epsilon$, and save ϵ of good 0 instead. By concavity $mu_h^i(\tilde{x}_h^i) < mu_h^i(\hat{x}_h^i) \leq p_h$, while the numeraire good has marginal utility of 1.²⁰ Thus, $MRS_{h,0}^i(\tilde{x}^i) = \frac{mu_h^i(\tilde{x}_h^i)}{mu_0^i(\tilde{x}_0^i)} < \frac{p_h}{1}$ and hence \tilde{x} violates voluntariness (V).

Suppose second $\tilde{x}_h^i < \hat{x}_h^i$, i.e. i receives less than desired. Then he is constrained in market h , $\hat{x}_h^i > \tilde{x}_h^i = \bar{Z}_h^i$ (the last equality follows from feasibility and voluntariness). By concavity $mu_h^i(\tilde{x}_h^i) > mu_h^i(\hat{x}_h^i) \geq p_h$ ($\hat{x}_h^i = 0$ is not possible since $\tilde{x}_h^i < \hat{x}_h^i$), while the numeraire good has marginal utility of 1. Since $\sum_{i \in N} \tilde{x}_h^i = \omega_h^h > \sum_{i \in N} \hat{x}_h^i$ (the inequality is due to the fact that h is a non-scarce good), there must be an agent j with $\tilde{x}_h^j > \hat{x}_h^j$. If $j \neq h$, then \tilde{x} violates voluntariness with respect to agent j as shown above (for agent i). Hence, consider the case that $j = h$. $\tilde{x}_h^h > \hat{x}_h^h$ means that the seller sells less than desired because $mu_h^h(\tilde{x}_h^h) < mu_h^h(\hat{x}_h^h) \leq p_h$ by concavity. Thus, $\hat{x}_h^h - \omega_h^h < \tilde{x}_h^h - \omega_h^h = \underline{Z}_h^h$ (the last equality follows from feasibility and voluntariness), i.e. the seller is constrained from selling more. This is a violation of weak order (WO). Indeed for t such that $t_h^i = \tilde{x}_h^i + \epsilon$ and $t_h^h = \omega_h^h - \tilde{x}_h^h - \epsilon$ and $t_0^i = \tilde{x}_0^i - \epsilon$ and $t_h^h = \tilde{x}_h^h + \epsilon$ and otherwise t fully corresponding to \tilde{x} , we have $t^i \succ^i t^i$ and $t^h \succ^h t^h$ and $(t_h^i - \bar{Z}_h^i)(t_h^h - \underline{Z}_h^h) = \epsilon \cdot (-\epsilon) < 0$.

- (b) Scarce good h : Consider an allocation \tilde{x} that does not satisfy this property. Suppose first that for some $i \neq h$, $\tilde{x}_h^i > \hat{x}_h^i$. This is a violation of voluntariness (V) as shown in the proof above.²¹ From now on assume that $\forall i \neq h$, $\tilde{x}_h^i \leq \hat{x}_h^i$ and $\tilde{x}_h^h \neq \hat{x}_h^h$.

Suppose first $\tilde{x}_h^h < \hat{x}_h^h$, i.e. h sells more than desired. Then $\underline{Z}_h^h \leq \tilde{x}_h^h - \omega_h^h < \hat{x}_h^h - \omega_h^h \leq 0 \leq \bar{Z}_h^h$. Hence, within the constraints and within h 's budget set, h could also reduce the amount that she sells from her good h and consume more herself, $\tilde{x}_h^h + \epsilon$, in exchange for a smaller amount of good h . By concavity $mu_h^h(\tilde{x}_h^h) > mu_h^h(\hat{x}_h^h) \geq p_h$, while the numeraire good has marginal utility of 1. Thus, \tilde{x} violates voluntariness (V).

Suppose second $\tilde{x}_h^h > \hat{x}_h^h$, i.e. h sells less than desired. Then she is constrained in

²⁰Boundary solutions are covered by " \leq ": $\hat{x}_h^i = 0$ is possible, but $\hat{x}_h^i = \omega_h^i$ not since $\hat{x}_h^i < \tilde{x}_h^i \leq \omega_h^i$.

²¹Indeed, then $\underline{Z}_h^i \leq 0 \leq \hat{x}_h^i < \tilde{x}_h^i \leq \bar{Z}_h^i$. Hence, within the constraints and within i 's budget set, i could also reduce the amount that he buys from good h , $\tilde{x}_h^i - \epsilon$ and save ϵ of good 0 instead. By concavity $mu_h^i(\tilde{x}_h^i) < mu_h^i(\hat{x}_h^i) \leq p_h$, while the numeraire good has marginal utility of 1.

market h , i.e. $\hat{x}_h^h - \omega_h^h < \tilde{x}_h^h - \omega_h^h = \underline{Z}_h^h$ (the last equality follows from feasibility and voluntariness). By concavity $mu_h^h(\tilde{x}_h^h) < mu_h^h(\hat{x}_h^h) \leq p_h$, while the numeraire good has marginal utility of 1. Since $\sum_{i \in N} \tilde{x}_h^i = \omega_h^h \leq \sum_{i \in N} \hat{x}_h^i$ (the inequality is due to the fact that h is a scarce good), there must be an agent i with $\tilde{x}_h^i < \hat{x}_h^i$, i.e. who buys less than desired. By concavity $mu_h^i(\tilde{x}_h^i) > mu_h^i(\hat{x}_h^i) \geq p_h$. Thus, (by feasibility and voluntariness) $\bar{Z}_h^i = \tilde{x}_h^i < \hat{x}_h^i$, i.e. buyer i is constrained from buying more. This is a violation of weak order (WO). Indeed for t such that $t_h^i = \tilde{x}_h^i + \epsilon$ and $t_h^h = \omega_h^h - \tilde{x}_h^h - \epsilon$ and $t_0^i = \tilde{x}_0^i - \epsilon$ and $t_h^h = \tilde{x}_h^h + \epsilon$ and otherwise t fully corresponding to \tilde{x} , we have $t^i \succ^i t^i$ and $t^h \succ^h t^h$ and $(t_h^i - \bar{Z}_h^i)(t_h^h - \underline{Z}_h^h) = \epsilon \cdot (-\epsilon) < 0$.

□

A.4 Proof of Proposition 2

Proof. There are two assertions to prove.

1. Pareto efficiency: Suppose good h is non-scarce and $\hat{x}_h^i > 0$ where $i \neq h$. Proposition 1 directly implies that in any FPE x : $MRS_{h,0}^h(x^i) < p_h$ and $MRS_{h,0}^i(x^i) = p_h$. Since preferences are continuous, a Pareto improving trade, in which h sells some amount to i at a price slightly below p_h , must exist.
2. Constrained efficiency: By weak interiority, the number of non-scarce markets is not equal to one.
 - (a) Suppose the number of non-scarce markets is larger than one. Take any supplier i of a non-scarce good i . By Proposition 1, in equilibrium $x_i^i > \hat{x}_i^i$ and hence $mu_i^i(x_i^i) < p_i$. By assumption of weak interiority, there exists another non-scarce good h such that $\hat{x}_h^i > 0$, which implies that in equilibrium $mu_h^i(x_h^i) \geq p_h$. Taken together $hi \in R^i$, where the binary relation R^i is defined for a fixed allocation x and fixed prices p_j and p_k as follows: $jk \in R^i \Leftrightarrow x_k^i > 0$ and $MRS_{j,k}^i(x^i) > \frac{p_j}{p_k}$.²² Denote $i = h_1$ and $h = h_2$. Since h_2 is non-scarce either, a good h_3 exists, such that $h_3 h_2 \in R^{h_2}$. If $h_3 = h_1$, a Pareto improving chain exists. If $h_3 \neq h_1$, a good h_4 must exist such that $h_4 h_3 \in R^{h_3}$. If $h_4 = h_1$ or $h_4 = h_2$, a Pareto improving chain exists. If not, there must be a good h_5 , and so on. Eventually at good h_{k+1} it must be that $h_{k+1} = h_1$ or $h_{k+1} = h_2$ or... or $h_{k+1} = h_{k-1}$; and we have found a Pareto improving chain.

²²The binary relation R^i indicates which trades agent i would accept. $jk \in R^i$ has the interpretation that agent i is willing to give up a small amount of good k to receive $\frac{p_k}{p_j}$ times that amount of good j .

(b) Suppose the number of non-scarce markets is zero. Take any market $h \neq 0$. The assumption $p_h^* \neq p_h$ implies $p_h^* > p_h$ for scarce goods (by Lemma 1). Since markets clear in Walrasian equilibrium and Walrasian prices are larger than fixed prices, there is at least one agent who is constrained from buying on this market. Hence, for each good $h \neq 0$, there is some agent i with $mu_h^i(x_h^i) > p_h$, while $mu_i^i(x_i^i) = p_i$ (by Proposition 1). Now, consider any good h_1 . By the argument above, there exists a good h_2 such that $mu_{h_1}^{h_2}(x_{h_1}^{h_2}) > p_{h_1}$, while $mu_{h_2}^{h_2}(x_{h_2}^{h_2}) = p_{h_2}$. Thus, $h_1 h_2 \in R^{h_2}$. Likewise, for good h_2 , there is an agent h_3 and the corresponding good h_3 such that $mu_{h_2}^{h_3}(x_{h_2}^{h_3}) > p_{h_2}$, while $mu_{h_3}^{h_3}(x_{h_3}^{h_3}) = p_{h_3}$. Thus, $h_2 h_3 \in R^{h_3}$. If $h_1 = h_3$, a Pareto improving chain exists. If $h_1 \neq h_3$, a good h_4 exists $mu_{h_3}^{h_4}(x_{h_3}^{h_4}) > p_{h_3}$, while $mu_{h_4}^{h_4}(x_{h_4}^{h_4}) = p_{h_4}$. Thus, $h_3 h_4 \in R^{h_4}$. If $h_4 = h_1$ or $h_4 = h_2$, a Pareto improving chain exists. If not, there must be a good h_5 , and so on. Since there are n such goods, eventually at good h_{n+1} it must be that $h_{n+1} = h_1$ or $h_{n+1} = h_2$ or... or $h_{n+1} = h_{n-1}$; and we have found a Pareto improving chain. □

A.5 Proof of Proposition 3

There are two assertions to prove.

1. Suppose good h is scarce. By Lemma 1, $p_h^* \geq p_h$. Hence, the demand of agent h for her own good is lower under Walrasian prices than under fixed prices. She gets her optimal amount of good h under Walrasian prices, but also under fixed prices since the good is scarce (by Proposition 1). Hence, $x_h^{h,*} \leq \hat{x}_h^h = x_h^h$. Thus, $\omega_h^h - \sum_{i \neq h} t_h^{i,*} = x_h^{h,*} \leq x_h^h = \omega_h^h - \sum_{i \neq h} t_h^i$, which yields the result.
2. Suppose h is non-scarce. By Lemma 1, $p_h^* \leq p_h$. Hence, the demand of all agents $i \neq h$ is larger under Walrasian prices than under fixed prices. Any agent $i \neq h$ gets her optimal amount of good h under Walrasian prices, but also under fixed prices since the good is non-scarce (by Proposition 1). Hence, $x_h^{i,*} \geq \hat{x}_h^i = x_h^i$. Thus, $t_h^i = x_h^i \leq x_h^{i,*} = t_h^{i,*}$, $\forall i \neq h$. □

A.6 Proof of Corollary 1

We first show that, under Assumption 1 (i), suppliers of scarce goods are earning more than suppliers of non-scarce goods in any FPE. We then show that income

increases for suppliers of scarce goods and, under Assumption 1 (ii), decreases for suppliers of non-scarce goods. For easier reference, we partition the set of agents into suppliers of scarce goods (SC) and suppliers of non-scarce goods (NSC). Let $i \in SC$ and $j \in NSC$ be two generic suppliers of scarce goods and non-scarce goods, respectively. By Proposition 1 the income of each supplier of a non-scarce good in a FPE is $y^j = \sum_{k \neq j} \hat{x}_j^k \cdot p_j$. By Prop. 1 the income of each supplier of a scarce good in a FPE is $y^i = (\omega_i^i - \hat{x}_i^i) \cdot p_i$. Assumption 1 (i), i.e. $\frac{Q_j}{S_i} < \frac{p_i}{p_j}$, can be written as $(\sum_{k \neq j} \hat{x}_j^k) p_j < (\omega_i^i - \hat{x}_i^i) p_i$, which then directly implies $y^i > y^j$, i.e. suppliers of scarce goods receive a higher income than suppliers of a non-scarce good.

We now show that $y^{i,*} > y^i$ for $i \in SC$. $y^{i,*} = |t_i^{i,*}| \cdot p_i^* > |t_i^i| \cdot p_i = y^i$ since by Proposition 3 $|t_i^{i,*}| > |t_i^i|$ and by Lemma 1 $p_i^* > p_i$.

Finally, we use Assumption 1 (ii) to show that $y^{j,*} < y^j$ for $j \in NSC$. We first rewrite $\epsilon_j^D = \frac{Q_j^* - Q_j}{p_j^* - p_j} \cdot \frac{p_j}{Q_j}$ to have $Q_j^* = Q_j(1 + \frac{p_j^* - p_j}{p_j} \epsilon_j^D)$, which we plug into the following expression.

$$y^{j,*} - y^j < 0 \quad (\text{A.1})$$

$$Q_j^* p_j^* - Q_j p_j < 0 \quad (\text{A.2})$$

$$Q_j(1 + \frac{p_j^* - p_j}{p_j} \epsilon_j^D) p_j^* - Q_j p_j < 0 \quad (\text{A.3})$$

$$Q_j \left[p_j^* + \frac{p_j^* - p_j}{p_j} \epsilon_j^D p_j^* - p_j \right] < 0 \quad (\text{A.4})$$

$$Q_j \left[(p_j^* - p_j) \left(1 + \frac{p_j^*}{p_j} \epsilon_j^D \right) \right] < 0 \quad (\text{A.5})$$

$Q_j > 0$ by assumption. By Lemma 1 we have $p_j^* - p_j < 0$. The elasticity ϵ_j^D is negative, but bounded from below by Assumption 1 (ii): $\epsilon_j^D \geq -1$. Since $\frac{p_j^*}{p_j} < 1$ (by Lemma 1), we have $\frac{p_j^*}{p_j} \epsilon_j^D > -1$ and $1 + \frac{p_j^*}{p_j} \epsilon_j^D > 0$. Therefore, the inequality holds. \square

B Extensions

B.1 More General Endowment

We briefly discuss how our results change when we relax the assumption on the endowments, i.e. that every agent is endowed with only one good and that the number of goods m must equal the number of agents n . Hence, there can now be many sellers of a good and an agent can sell many goods. We call every agent who

is endowed with more than he desires, i.e. $\omega_h^j > \hat{x}_h^j$, net supplier of this good and all others net demanders. Then the characterization of all FPE becomes:

Proposition B.1 (General Characterization). *In every FPE, each good $h \neq 0$ is allocated as follows:*

1. *If h is non-scarce, every net demander receives the desired amount, while every net supplier receives at least the desired amount. That is: $\forall i$ with $\omega_h^i \leq \hat{x}_h^i$, we have $x_h^i = \hat{x}_h^i$; and $\forall j$ with $\omega_h^j > \hat{x}_h^j$, we have $x_h^j \geq \hat{x}_h^j$.*
2. *If h is scarce, every net demander receives at most his desired amount, while the net suppliers keep (exactly) the desired amount. That is: $\forall i$ with $\omega_h^i \leq \hat{x}_h^i$, we have $x_h^i \leq \hat{x}_h^i$; and $\forall j$ with $\omega_h^j > \hat{x}_h^j$, we have $x_h^j = \hat{x}_h^j$.*

Proof. The proof is fully analogous to the proof of Proposition 1. □

As Proposition B.1 shows, the characterization of Proposition 1 generalizes to the set-up with more general endowments. Only the statement about net suppliers of non-scarce goods becomes weaker. Before, the excess supply was kept by the unique seller. Now, the notion of FPE does not determine how the excess supply is allocated among the net suppliers. The other parts are identical to Proposition 1.

For the results on inefficiency (Proposition 2) and less trade (Proposition 3) this leads to some adaptations but does not change the substance.

B.2 More General Preferences

In this section, we extend the model by relaxing the assumption that the utility function is quasi-linear. The more general utility function has the following form:

$$U^i(x^i) = u_0^i(x_0^i) + u_1^i(x_1^i) + \dots + u_h^i(x_h^i) + \dots + u_n^i(x_n^i),$$

with marginal utility $mu_h^i(x_h^i) > 0$ and $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} \leq 0$ for all i, h and x_h^i ; the inequality $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} \leq 0$ is strict for all $h \neq 0$. A simple characterization as in Proposition 1 is then no longer possible because demand and supply on each market may now depend on the allocation on all other markets. It is even possible that a scarce good “becomes non-scarce” in the sense that there is excess supply in the fixed price equilibrium; and vice versa. Since Proposition 1 was key to show inefficiency (Proposition 2), the question arises whether this result can be reestablished. The short answer is: yes, partially.

We can show first that for each scarce good i there must exist an agent j who would be willing to trade good i in exchange for his own good j (but not necessarily for good 0).

Lemma B.1. *If good h is strictly scarce, i.e. $\sum_{i \in N} \hat{x}_h^i > \omega_h^h$, then in any FPE x there is an agent j who would like to trade h in exchange for his own good, i.e. $MRS_{h,j}^j(x^j) > \frac{p_h}{p_j}$.*

Proof. We first show that the seller of the scarce good h , receives at least the desired amount, i.e. $x_h^h \geq \hat{x}_h^h$. Assume not such that $x_h^h < \hat{x}_h^h$. By voluntariness (V), we then have $x_0^h < \hat{x}_0^h$. Again by voluntariness (V), this implies $x_k^h < \hat{x}_k^h$ for any good k . Thus, $x_h^h < \hat{x}_h^h$ implies $x_k^h < \hat{x}_k^h$ for any good k (including the numeraire). But then $px < p\omega$. Hence, x cannot be an equilibrium allocation. Second, if h is strictly scarce, there must be an agent j such that $mu_h^j(\hat{x}_h^j) < mu_h^j(x_h^j)$. Together, we therefore have $p_h mu_j^j(x_h^j) \leq p_h mu_j^j(\hat{x}_h^j) = p_j mu_h^j(\hat{x}_h^j) < p_j mu_h^j(x_h^j)$. \square

Lemma B.1 can be interpreted as follows: every (initially) scarce good remains “somewhat scarce.” The main reason is that quantity constraints on the demand side can never increase supply. Hence, if there are two agents i and j , who both have a larger demand for the other’s good than the other’s (unconstrained) supply is, then they could improve in each FPE by mutual trade at the given price scheme. This leads to one kind of inefficiency that we establish in the following extension of Proposition 2.

Proposition B.2. *If there is a set of agents S such that their demand for their own goods exceeds the endowment, i.e. $\forall i, h \in S, \sum_{i \in S} \hat{x}_h^i > \omega_h^h$, then no FPE is constrained efficient.*

Proof. From Lemma B.1 we know that $\forall h \in S, x_h^h \geq \hat{x}_h^h$. Thus, for some $i \in S$, $x_h^i < \hat{x}_h^i$. This directly implies $hi \in R^i$ because $mu_h^i(\hat{x}_h^i) = \frac{p_h}{p_i} mu_i^i(\hat{x}_h^i)$ and $x_h^i \geq \hat{x}_h^i$ (again from Lemma B.1). At the same time there must exist an agent $j \neq i \in S$ such that $x_h^j < \hat{x}_h^j$. For the same reason as above $ij \in R^j$. We can continue as in the proof for Proposition 2 until we have found a Pareto improving chain. \square

Hence, fixed prices often lead to constrained inefficient allocations even with more general preferences. We have shown this for one type of inefficiency (scarce goods, prices are “too low”), while for another (non-scarce goods, prices are “too high”) the analogous result cannot be established. The reasons is that quantity constraints on the demand side can easily increase demand for other goods. Hence, our inefficiency result, Proposition 2 partially extends to more general preferences.

Importantly, the effects isolated in the special case of quasi-linear preferences are still at work, they are in general simply accompanied by other potential effects.

C Appendix: Additional Figures

C.1 Lorenz Curves

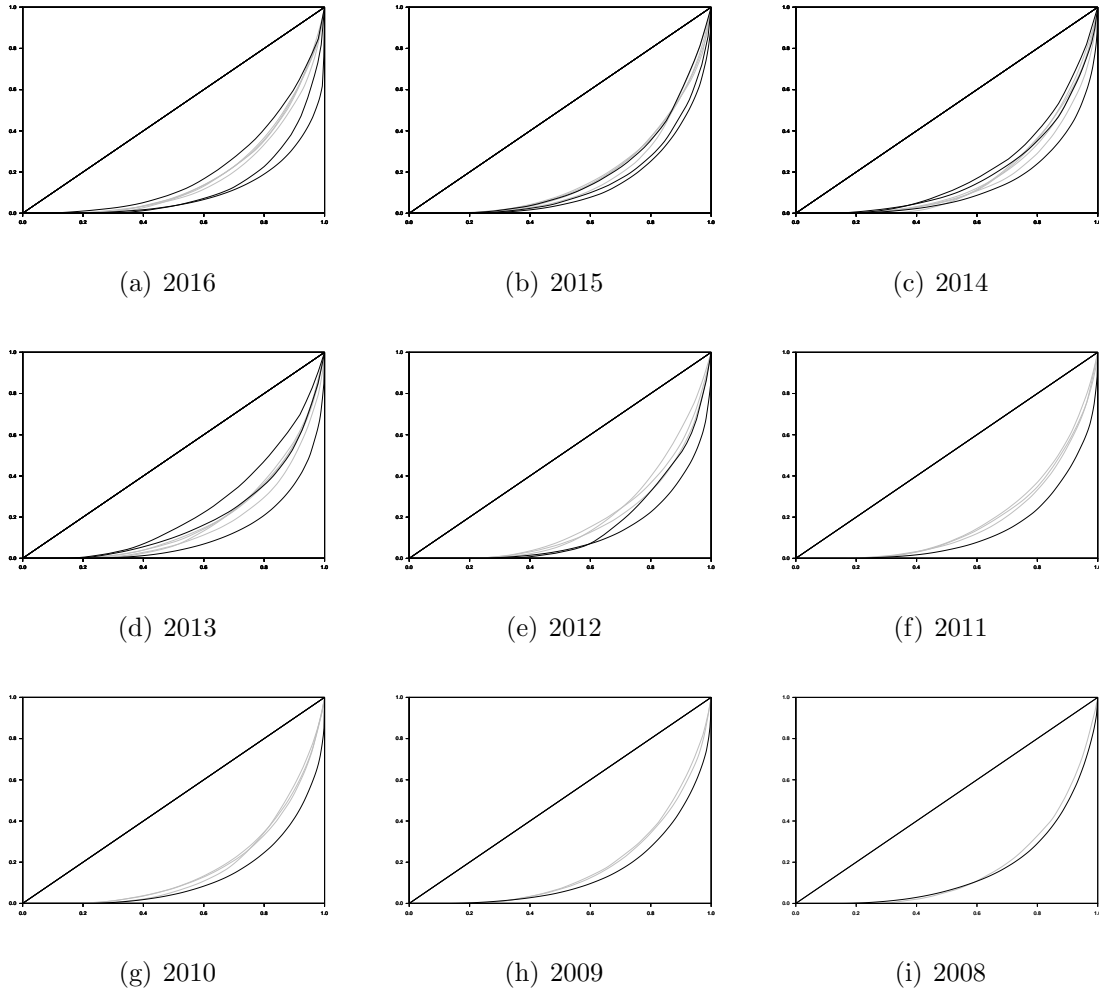


Figure C.1: Lorenz curves of income distribution for each platform by year. The id line is the benchmark of full equality. The other black lines illustrate inequality of platforms with flexible prices W1-W3 and the red (or gray) lines of those with fixed prices F1-F4. Lorenz domination is visible when one line fully lies below another line.

References

- ALESINA, A., AND G.-M. ANGELETOS (2005): “Fairness and Redistribution,” *The American Economic Review*, 95(4), 960–980.
- ALMÁS, I., A. W. CAPPELEN, E. Ø. SØRENSEN, AND B. TUNGODDEN (2010): “Fairness and the Development of Inequality Acceptance,” *Science*, 328(5982), 1176–1178.
- BÉNABOU, R., AND J. TIROLE (2006): “Belief in a Just World and Redistributive Politics,” *The Quarterly Journal of Economics*, 121(2), 699–746.
- CAHN, E. (2011): “Time Banking: An Idea Whose Time Has Come?,” *Yes Magazine*, Retrieved 22 March 2018.
- COWELL, F. A. (2000): “Measurement of inequality,” *Handbook of Income Distribution*, 1, 87–166.
- FOSTER, J. E. (1983): “An axiomatic characterization of the Theil measure of income inequality,” *Journal of Economic Theory*, 31(1), 105–121.
- GRANDMONT, J.-M. (1977): “The logic of the fix-price method,” *The Scandinavian Journal of Economics*, pp. 169–186.
- HERINGS, P. J.-J., AND A. KONOVALOV (2009): “Constrained suboptimality when prices are non-competitive,” *Journal of Mathematical Economics*, 45(1), 43–58.
- MAILATH, G. J., A. POSTLEWAITE, AND L. SAMUELSON (2016): “Buying Locally,” *International Economic Review*, 57(4), 1179–1200.
- MASKIN, E. S., AND J. TIROLE (1984): “On the efficiency of fixed price equilibrium,” *Journal of Economic Theory*, 32(2), 317–327.
- WARREN, J. (1852): *Equitable commerce: A new development of principles*. New York: Burt Franklin Press.
- YOUNÉS, Y. (1975): “On the Role of Money in the Process of Exchange and the Existence of a Non-Walrasian Equilibrium,” *The Review of Economic Studies*, 42(4), 489–501.