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I.G. Tuluzov[1], S.I.Melnyk[2]

[1]*Kharkiv Regional Center for Investment, Kharkiv, Ukraine*
[2]*O. Ya. Usikov Institute for Radiophysics and Electronics of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine*

SOLUTION OF PROBLEMS OF DYNAMICS IN THE INFORMATION SPACE OF STATES ON THE BASIS OF THE PRINCIPLE OF COMPLEXITY MINIMIZATION

The principle of minimizing of the complexity allows us to introduce a quantitative measure of information in algorithmic information theory. It is similar to the physical principle of least action in the vector space of the information states. The variational problem of finding the optimal description of the information object "B" for a given object "A" corresponds to the determination of the classical trajectory of a material point for a given initial and final position in the physical space-time. At the same time, the physical in the own reference system is equivalent to the length of the suffix of the compressed binary word. The proposed analogies of the information and physical theories simplify the description of the complex systems of information objects.

Keywords: *information space, the dynamics, the principle of least action, relativism, fundamental measurement, invariance, the theory of complexity*

Introduction

The most common formulation of the problem of information theory is the prediction of the results of the yet performed measurements. This is done based on the information that has already been obtained in previous measurements. However, the existing classical approaches to its construction based on the theory of sets, classical logic, probability theory and other mathematical structures, not associated with the measurements. In this regard, to answer the question further build a physical model of the observed object or phenomenon, and information theory is used only as an auxiliary language to describe the physical properties specified in the conditions of incomplete information about the results of the measurements. Such an approach leads to many contradictions and paradoxes, which have been discussed in the literature.

The report submitted at this conference, we have shown that the methods of the algorithmic theory of information (the theory of "complexity") allow us to introduce a partial ordering on the set of data objects, and the invariance of the requirements for the selection of descriptive language to consider them as elements of the relativistic vector space. This information naturally arise concepts of information time and information distance in such a space. This opens up a new opportunity not only to consider the results of fundamental measurement as the primary, but also to build a model of the observed object on the basis of them. In particular, the set of events "happening" in the information space, have only an indirect relationship to the events of the physical world. But they describe operators of transition between the pairs of the information states in the information space-time.

It is natural to expect that in this case other laws interconnection of different information states can also be described in the spirit of the physical paradigm. In particular, they can be represented by the variational principle, the same principle of least action. This information analogue of the function will be determined by the symmetry properties of the information space constructed before, as in physics. Previously, we have developed an analog of physical space-time for the partially ordered set of economic states and built the foundations of the dynamics in this space [1]. This work is the first attempt to trace the possibility of such a path for building dynamics in the information space-time.

Information «clock»

A scale of ordering of elements of a set, analogous to the scale of physical time, can be constructed in the theory of information as well. Formally, any sequence of information objected ordered by the relation of inclusion can be considered as such scale. Unlike the physical clock, it does not require a pair of mirrors and a light signal if a different method of calculation of the absolute and relative information contained in these objects is set. In this case, we can associate each element of the sequence with a number equal in bits to the quantity of information contained in it. Moreover, discreteness of the information scale (measurement of the “information time” in bits) is natural and does not require a separate substantiation. With such definition, there is no necessity in the synchronization of the information clock, as the value of one bit in the classical theory of information is absolute and does not depend on the selection the language of description.

Thus, ***the information analog of time interval*** between two events in the physical space separated by a time-like interval is the quantity of information required for the transition from the information object A to the information object B. In other words, it is the length of the program (in bits), in which the object A is set at the input and the object B is obtained at the output. Let us note that in physics the time interval between two events depends on the reference system in which the measurements are performed (trajectories of clock motion). In the theory of information the corresponding quantity of information depends on the programming language, which can be considered the ***analog of the reference system.*** Different chains of information states, each of which include all the previous ones, correspond to different trajectories of the observer's clock in physics. Next, we confine our attention to a narrow class of programming languages, described by the binary alphabet. In the appropriate spaces trajectory (the program of the transition between data objects) are binary sequences ("words"). But at the same time we require their equality and the lack of highlighted information "a frame of reference".

Two approaches to the description of the procedure of "compression" of information

The illusion associated with the ideas of the existence of a "world ether", taken as the selected frame of reference in physics led to a number of contradictions that have been successfully resolved in the framework of the relativistic theory. We show that the requirement of invariance binary description languages leads to a similar result. Consider the geometric representation of words in the two-dimensional discrete space. Each binary word we associate a broken line.

Moreover, "0" corresponds a step to the left-up, and "1" - a step to the right-up. Coordinate "word" on the vertical axis corresponds to the number of characters in the already recorded portion "words", and the horizontal - the difference in the number of "1" and "0" in a word. When recording the first coordinate words can only increase, while the second - to change on both sides. Therefore, we now note the analogy of the model constructed with the structure of the one-dimensional space-time physics, and we denote the dimension of this space as a 1 + 1.

Each binary word written in some language, we can associate a path (a broken line). Some of the binary words can be written shorter ("compress"). It is sufficient that the frequency deviation of the "0" on some part of the trajectory different from ½ on a statistically significant amount. To describe the operation of compression (encoding) the word in the information space, there may be at least two ways:

• It is possible to assume that the words compression operation transforms it into this space to another binary word with fewer characters.

• You can reduce the complexity of the description of the original word by going to a different language to describe (the letters «a» and «b», for example).

Fig. 1 (a) illustrates the procedure of compression of the word

"A" = "110110110111101111"

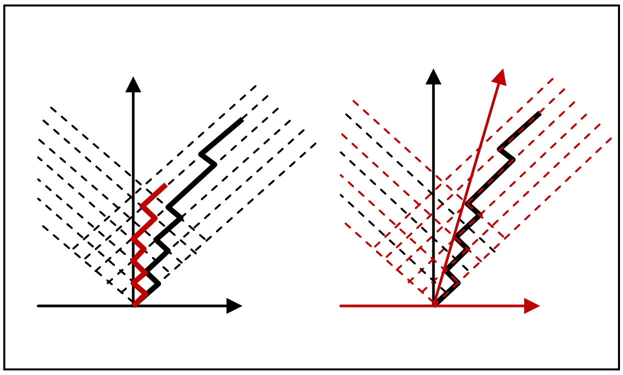
into the new word

"A *" = "10101011011"

of the same language, as in Figure 1 (b) - the translation of the same word to description of new language

«110110110111101111» = «abababaabaa».

At the same time, as it can be seen from the figure, according to the new language changes the direction of the coordinate axes, and the length of the vectors corresponding to the new "letters" of the alphabet. Technically, both new forms of recording the words "A" are equivalent (they differ only in the symbol designation), the second method, we believe a more preferred. It is connected with the idea of information as some objective fact, which ideally should not depend on the language in which it is described. Therefore, in the information space as the end position and intermediate points in the trajectory of the word, should not be changed by replacing the language to describe it. The physical theory is used from the beginning, only the second method of describing trajectories. Otherwise, each time turning her head, we would consider that others change their body position.



(a) *(b)*

Fig.1. Comparison of two ways of representing the original (black line) and compressed (red line) words.

Physical analog of the "prefix" and "suffix" of the compressed binary word

As a result of compression of the binary words of one of the standard methods, all information contained in it, is divided into two fundamentally different parts. The first of them (the prefix) contains a description of all those laws, which make it possible to reduce the recording word. The remainder of the compressed random words contained in strictly algorithmic sense information. This means that there is not any other (not yet recorded) laws that allow to write the compressed word even shorter [3].Let us consider the physical analogy of regular and random parts of information contained in a compressed notation of a binary word. Let us assume, for instance, that it is required to describe the classical trajectory of a material point as a result of measurements performed using the Einstein's clock. Then this description will be represented as a sequence of calculated coordinates of reflection of consecutive light signals from the surface of the observed body. Discreteness of such representation depends on the distance between the mirrors and the obtained trajectory of motion appears to be a binary "word". In this word we can single out the regular part – the frequency "1", and the random part – their location in the sequence. The regular part is defined by the relative velocity of motion of the body $w_1(t) = 1/2 \cdot (1 - v/c)$, while the random part is defined by the distance between the mirrors and their exact location.

Let us note that setting only the regular part of the binary word $w_1(n)$ unambiguously defines the whole trajectory in any reference system. Based on this analogy, we will further consider the regular part of the compressed binary word ("prefix") as the objective part of complete information contained in this word, and the random part ("suffix") – as the subjective. At the same time the number n of each symbol of the binary word corresponds to the time (number of discrete "tic-tacs") in the reference system corresponding to the selected language of description. In the "suffix" of the maximally compressed word $w_1(n) = 1/2$. It means that the symbols of such language of description correspond to the intrinsic frame of reference (FR), in which the body is static $v(t) = 0$. In the any others FR $w_1(n) \neq 1/2$ and the length of notation of the "suffix" increases according to the Shannon's formula. It is completer analogous to the effect of relativistic dilation of physical time in moving reference systems. It can be shown that the formulas of transformation of the "prefix" $w_1(n)$ and the length of the "suffix" for the set of invariant binary languages are equivalent to Lorentz's transformations in the special theory of relativity. There is nothing surprising in it, as these formulas are derived using the same principles of invariance. Thus, the rest of the laws of **kinematics** of the relativistic (invariant) space of binary languages of description can also be obtained. However, the solution of problems of information dynamics requires using additional principles.

Information analog of the principle of least action

The problem of classical dynamics in its simplest form is to select the trajectory of motion of bodies in the expanded coordinate space by setting its start and end positions. To solve this problem, the variational method minimizes the action function, whose form is determined by the properties of the observed system of bodies and external force fields, which are defined as a function of time and coordinates (its degrees of freedom). It is shown that in relativistic mechanics $S_{AB} = -m\int_A^B ds$, where ds - the distance between adjacent points trajectory. This integral corresponds to the length of time between its endpoints measured in its own frame of reference. And with the sign "-" a minimum of S_{AB} provides maximum proper time. In other words, in the relativistic mechanics a moving body freely "choose" the path of movement, to which his own "clock" will show the maximum time [4].

If we consider the classical trajectory of a binary word, it appears that the length of the "suffix" of the true trajectory is maximum, compared with all the other paths. At the first glance, this result contradicts the main paradigm of the algebraic theory of information – the principle of minimization of complexity of description (search of a program of minimal length which converts the information object "A" into "B"). However, in case of a more detailed analysis we can solve this contradiction.

The point is that both in physical and in information space we are dealing with two different problems of optimization instead of one. In the first problem (selection of the trajectory of motion) we maximize the intrinsic time for the true trajectory. In the second – we find its simplest description transferring into the intrinsic reference system. In this case, the time calculated in the intrinsic reference system turns out to be minimal compared to the rest of the systems.

The first variation problem (problem of classical dynamics) is to maximize the length of the "suffix" in the optimum trajectory record. It means that such a trajectory (program of transition from A to B) is to be found, in which the minimal quantity of regularities is present. Actually, these regularities contain only the a priori information on the finite points of the trajectory and its continuity. This requirement can be associated with the principle of the "Occam's razor". In the found trajectories there are no "excess" economic entities, not following from the problem statement. This result also corresponds in the generalized sense to the principle of maximum likelihood. In our paper [5] we have shown that it can be obtained by means of the "theory of complexity". The obtained trajectory allows maximal quantity of random realizations and therefore can be considered maximally plausible.

In the ***second variation problem*** we no longer change the obtained trajectory; we are to find the simplest method of description of each of its random realizations. For this purpose, all its regularities set by the dependence $w_1(n)$, are transferred into the prefix, while the length of the suffix turns out to be minimal in a set of various languages of description. It is essential that the length of the random part of the optimal trajectory (information analog of action) can also be calculated for any other language of description. However, for a different language it will no longer be equal to the time interval between the two events.

Similarly to the physical space, we can introduce the notion of information interval between two information objects. Thus, the value of this invariant can be considered as a certain "absolute" quantity of information separating of the information objects "A" and "B". Its value is invariant to the choice of description language (discussed above sets of equitable binary language).

Summing up this consideration, we can state the following:

- The problem of information dynamics is reduced to the determination of the most plausible trajectory of transition from one information object to another.
- At maximal compression (according to Vietinghoff, for instance) the description of these states consists of the "prefix" and the "suffix".
- By the trajectory of transition is meant the regular part of the program (its "prefix") written in one of the equal binary languages.

- The Lorentz's transformations in the information space allow transition from one language to another in the recording of the "prefix".
- The highest plausibility of the information trajectory corresponds to the maximal number of its random realizations or the maximal "suffix" length.
- This length is invariant to the selection of the language of description and is analogous to the value of the interval between time-like events in the physical space.
- The most plausible information trajectory is determined by solving the variation problem of maximization of the "suffix" length at the set initial and finite information states.
- This problem is equivalent to the principle of least action in the classical relativistic mechanics.
- The selection of language, in which the algorithmic complexity of the found trajectory is minimal, corresponds to the transition into the intrinsic reference system in physics.

Conclusion

By accepting the "physical" approach to the construction of the fundamental laws of the theory of information, we open a path for constructing it as a dynamic theory, in which, similarly to physics, the notions of space-time, forces, interaction appear. In general, this trend can be called "information physics", similarly to the term "econophysics" used in similar situations in economics. Analysis of the history of development of the physical theory allows us to predict further perspectives of the development of "information physics". Expansion of the set of invariant languages of description in such a way that it would include not only the languages corresponding to the inertial motion, but also the "non-inertial" ones, will likely result in the origin of an information analog of the general theory of relativity. Further expansion of the types of used symmetries is connected with the discreteness of representation of information objects, and, apparently, will require the use of the quantum-mechanical approach.

The question is – whether a finite number of fundamental symmetries which we need to consider for the description of all possible interactions exist, or we are "condemned" to introduce increasingly more complex symmetries for the description of more and more complex systems, remains open in physics as well [6]. In the present publication we have limited ourselves to the analysis of the initial stage of this chain.

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