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THE POSSIBILITY OF CONSTRUCTING OF THE INFORMATION QUANTUM-RELATIVISTIC SPACE OF STATES

Algorithmic theory of "complexity" is the most common approach to the description of the measurement results and their analysis. It has been shown that it allows us to introduce a partial ordering on the set of results of basic measurement in physics, information theory and economics. However, the properties of languages, which is calculated using the "complexity" is not symmetrical. The requirement to lack a dedicated description language allows us to represent data objects as elements of a relativistic vector space. The "transfer rules" from one language to another of the description of the program correspond to the coordinate transformations from one reference system to another in the physical space-time. Many events in this information space is not given a priori, and is formed as a representation of the results of fundamental measurement. This allows us to consider the problem of information theory similar to the objectives of the physical dynamics.

Keywords: *information space, relativism, fundamental measurement, invariance, theory of complexity*

Introduction

Measurements in physics are associated with the space-time continuum, which allows solving the problems of dynamics and optimizing the operation of natural and technical systems. In this connection, numerous attempts of describing other (primarily, social-economic) systems as elements of virtual vector space and limiting the problems of prediction and optimization of their behavior to the tasks of physical dynamics have been made.

There are two main obstacles in the direction. The first is connected with the necessity of formalizing of the notions of virtual distance and intervals of virtual time between a pair of different system states. The second is connected with the presence of a subjective element in the description of their properties. This obstacle can be overcome in the theory of fundamental measurements. Previously, the model of relativistic space of economic states has been represented in our paper [1] and it has been shown that its generalization is applicable for the analysis of states of a subject's consciousness [2]. At the same time, it appeared that the presence of a subjective component inevitably results in the necessity of application of the quantum-mechanical formalism for the description of the dynamics of such systems [3,4]. The obtained analogs between the behavior of physical and social-economic systems occur not as a result of some hidden physical mechanisms of functioning of the latter, but as a result of the general information-measurement approach to the construction of the vector space of their states.

In the present paper we will show that both physical and social-economic vector spaces are particular cases of the information space of the fundamental measurements and reflect all the properties and special features of the latter. By defining the fundamental measurement, we not only set the symmetries of the associated space, but also, to a significant degree, define the properties and laws of dynamics of the objects observed in this space.

Fundamental measurements in physics, in economics, and in the theory of information

We will consider the fundamental measurement as a procedure of comparison of two objects of a certain set and the result of such measurement – their ordering relation.

In the physical theory such property is exhibited by the relation of time ordering of any two events.

However, in physics, like in any other science, the aim is to extract the objective properties of the observed system invariant to the selection of the method of observation.

Thus, for any pair of events "A" and "B" one of the following three statements is valid:

- Event «A» for any observer occurs later than event "B" (is in the upper light cone in relation to it)
- Event "A" for any observer occurs earlier than event "B" (is in the lower light cone in relation to it)
- Events "A" and "B" are space-like and can have different time order for different observers

One of these three results will be referred to as the result of the absolute fundamental measurement in the physical theory.

By analogy with physics, we will define the absolute fundamental measurement **in the theory of information**. The result of the absolute fundamental measurement in the theory of information is one of the three statements:

- the information object "A" contains all the information contained in the information object "B";
- the information object "B" contains all the information contained in the information object "A";
- The information "A" and the information "B" are not completely contained in each other.

At the same time we assume that in the third case the object "A" can contain part of the information about the object "B" and vice verse.

In the economic theory the relation of partial ordering can be introduced on the basis of the procedure of transaction considered as a fundamental measurement. For any pair of economic objects "A" and "B" we can make one of the following three statements:

- Any of the proprietors will agree for a transaction on exchange of the object "A" possessed by him for the object "B" (object A is absolutely less expensive than object B)
- Any of the proprietors will reject a transaction on exchange of the object "A" for the object "B" (object A is absolutely more expensive than object B)
- Agreement or refusal of the transaction of the exchange "A" for "B" can depend on the proprietor's opinion.

The fundamental measurements in economics and the relativistic space of economic states constructed on the basis of their analysis have been described in detail in our papers [1, 2]. ***Thus***, we can see that the fundamental measurements in all these areas of knowledge are represented by equivalent mathematical structures.

Simple example of constructing the information space on the basis of a set of fundamental measurements

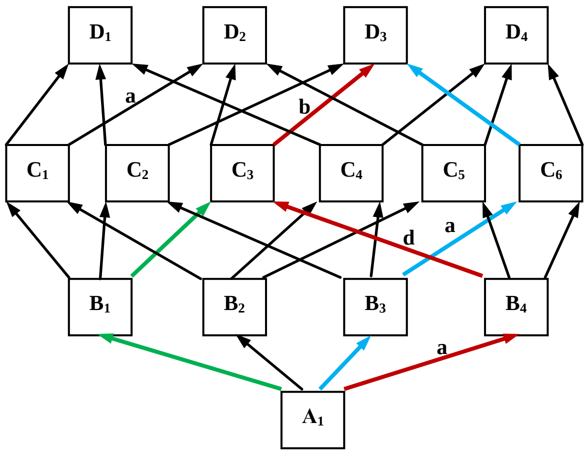
In classical logic any information object has an unambiguous representation in the form of a set of elementary (atomic) statements. For instance, in case of a single-time throwing of 2 coins the following elementary results are possible:

(a) - HH; (b) – HT; (c) – TH; (d) – TT

The rest of the statements (information objects) on the results of observation can be obtained from atomic statements using logical operations. Any of them can be associated with a subset of elementary results, for which it is true. And vice verse, for any subset of elementary results a corresponding fundamental measurement exists (statement, which can possess one of two values). In the discussed example the number of such different subsets is 15 (including the set of all elements). Their structure (according to the quantity of elements of the subset) can be set as (1+4+6+4). The graph of elementary edges for these information objects is illustrated in Fig.1.

In this graph a sequence of elementary edges (shown in red) can be arbitrarily selected and set as a "time" scale. Relative to this scale, the rest of the graph nodes are in an unequal position. Thus, for instance, the node «B_1» is at the "distance" of (1+1) edges from it, while the node «B_3» is at the "distance" of (1+2) edges. And if we will construct a kind of space, in which all of the node will have the corresponding coordinates, then they will be defined by this number of elementary edges connecting the mode with the "world line" taken as the time axis. We can state that the nodes «A_1»; «B_4»; «C_3»; «D_3» are at the origin of the coordinates and the rest of them are located at some distance from them. At the same time, the nodes «C_6» and «C_2» are characterized by an identical set (2+1), but contain different information. Therefore they must be located at the same distance, but in different points of the space. From this follows that even in such a simple example a one-dimensional space is insufficient for locating all graph nodes in it.

Any trajectory in the space of information spaces can be represented as a sequence of "events". For instance, the trajectory selected as a "world line" is described by the events **(a;b;d)**. At the same time, the same "event", for instance, **(a),** can occur in different points of the information space.



$$A_1 \equiv (a \cup b \cup c \cup d)\ B_1 \equiv (a \cup b \cup c)\ B_2 \equiv (a \cup b \cup d)$$
$$B_3 \equiv (a \cup c \cup d)\ B_4 \equiv (b \cup c \cup d)\ C_1 \equiv (a \cup b)$$
$$C_2 \equiv (a \cup c)\ C_3 \equiv (c \cup b)\ C_4 \equiv (a \cup d)\ C_5 \equiv (b \cup d)$$
$$C_6 \equiv (c \cup d)\ D_1 \equiv (a)\ D_2 \equiv (b)\ D_3 \equiv (c)\ D_4 \equiv (d)$$
$$E_1 \equiv (0)$$

Fig.1. Graph of elementary edges of absolute ordering of information objects and possible construction of a time scale in it.

We will not fix our attention on the analysis this simple example in detail, as it is very far from execution of axioms of the vector space. The aforesaid example should be considered only as an illustration of the possibility of constructing an information space of states. The space obtained in this process is neither vector, nor contiguous, though it has a specific symmetry imposed by the initial description.

In it we have a set of elementary outcomes of the observations, a priori defined by means of another language (heads - tails) and many fundamental measures, built on its basis with the help of logical connections.

In the framework of the information-measurement approach to the construction of the space of states we, first of all, answer the question on what properties should the information objects possess so that the set of fundamental measurements would form a vector space, which would be, at the same time, maximally symmetric.

Connection of the results of fundamental measurements with the relative complexity of description

The conditional complexity $S_{\Omega_k}(A/B)$ of the information object «B» relative to object «A» is referred to the as minimal length $]p_i[$ of program p_i, which translates «A» in «B».

$$S_{\Omega_k}(A/B) = \min]p_i[\ \ if \ \ B = p_i(A) \qquad (1)$$

As an example of such languages we can consider the Turing machine, Post machine and others.

If a result of absolute fundamental measurements $A \to B$ is obtained, it means that the information "A" is contained in the information "B" at any description (unambiguously follows from it). In the theory of complexity the quantity of information $K_S(A/B)$ about «A», contained in «B», is calculated as

$$K_S(A/B) = S_{\Omega_k}(A) - S_{\Omega_k}(A/B), \qquad (2)$$

where $S_{\Omega_k}(A)$ is the unconditional complexity of "A", which equals by definition to the quantity of information contained in "A". Then, however, in case of fulfillment of the condition $A \to B$ we obtain: $K_S(A/B) = S_{\Omega_k}(A)$, from which it follows that

$$S_{\Omega_k}(A/B) = 0 \qquad (3)$$

We will assume the complete description of a certain fundamental measurement "A" (a statement or a set of connected statements) as any other set of statements, which gives the same answers to all the questions, which can be obtained on the basis of the information "A". We will assume the conditional complexity of description of the fundamental measurement "A" in relation to "B" as the minimal quantity of results of fundamental measurements (information bit), which must be added to "B" in order to completely define "A" in the aforesaid sense.

Thus, if $A \to B$, then if "B" is set at the input of the program, we already have all the answers defining "A". Therefore, the conditional complexity of the description of "A" in relation to "B", the definition of which is adapted for the theory of fundamental measurements, equals 0. The difference from the classical definition of the conditional complexity is that we do not require the deletion of excessive information.

$$S^*_{\Omega_k}(A/B) = \min]p_i[\ \ if \forall k \ \ F_k(B) = F_k(p_i(A)) \qquad (4)$$

At the same time, the converse can appear to be false. Then the complexity of description of "B" in relation to "A" is > 0 and we consider that $A \to B$. If both relative complexities equal 0, then these are equivalent information objects (indivisible by any of the possible results of fundamental measurements).

Thus, the values of conditional complexity for a pair of objects "A" and "B" unambiguously define the relation of the absolute ordering (information inclusion) between them. However, the converse is also true. If it is known that $A \to B$, it means that the information object "B" sets unambiguous answers (Yes or No) for the same questions as "A" and additionally for some other questions. In case if there are more than one such additional answers, we can find such an object "C" that $A \to C \to B$. In the opposite case, the edge of the directed graph between "A" and "B" is an ***elementary edge*** As the reception of an answer for one additional elementary question (fundamental measurement) requires one bit of information, we will consider the conditional complexity of such objects equal to 1. Then the conditional complexity of two random objects, for which $A \to B$, can be defined as a minimal number of elementary edges in the chain $A \to C_1 \to C_2 \to C_3 ... \to B$. It is obvious that in this case the obtained value $S^*_{\Omega_k}(A/B)$ depends on the properties of the programming language Ω_k.

Let us note that for any elementary edge a ***conjugate elementary edge*** of the graph must exits, corresponding to the opposite answer to the same question. In the opposite case (absence of such alternative information object and the corresponding elementary edge) the question loses its sense.

Thus, we have managed to define the results of the absolute fundamental measurements through the values of conditional complexity and vice verse: calculate the values of the conditional complexity using the results of the absolute fundamental measurements. At the same time, using the absolute fundamental measurements, we have also defined the elementary operators of the used programming language. The action of these procedures on a certain information object causes its changing for one bit of information - possibility of receiving answer for one additional question (answers "yes" and "no" correspond to different operators). Moreover, we can totally ignore the representation of the programming language operators and the information objects in the framework of the used alphabets. Instead we can define both the operators and the information objects on the basis of the elementary edges that connect them.

A similar information-measurement approach to the definition of terms is used in any sufficiently full definition dictionary. Each word in it is described using other words, which are, in turn, also described in this dictionary. Thus, a rather perspicacious reader can understand the meaning of at least some of the words by analyzing their interconnections (analog of a graph of partial ordering).

«Trajectory» representation of the operators of the programming language and the information objects

In the general case, the alphabet of such language is finite and is different from a finite alphabet used for setting the information objects (Turing machine, for instance). In order reduce it to the ***space-trajectory representation*** we can assume the following sequence of procedures:

- Determining a set of information objects with which this language is operating (for instance, a set of finite binary sequences).
- Finding for each pair of these objects a corresponding value of the conditional complexity of description (for a fixed programming language it can be done exactly).
- Constructing a set of the results of absolute fundamental measurements on the basis of the values of relative complexity for each pair of objects and single out a set of elementary directed edges from them.
- Selecting one of the nodes of the obtained directed graph as reference point and the corresponding sequence of elementary edges as a time scale.
- Determining by some or other method the distance on the basis of the results of absolute fundamental measurements and the selected scale of information time.

As a result we obtain a directed graph analogous to the graph shown in Fig.1. However, for its construction we no longer need to introduce a set of elementary events and to associate each of the information objects to a particular subset. Instead, we will use the results of the absolute fundamental measurements, which are calculated independently. A natural question arises: "Is it possible to represent the nodes of the graph of information states (information objects) as all possible subsets of the set of elementary events?"

In some cases the answer to this question is given by the ***Stone's theorem*** [5]. It states that for any distributive structure a monomorphism exists representing it as a structure of all subsets of a certain set in such a way, that a complement transits into a complement. A partially ordered set can be considered a structure (grid) if for any pair of its elements an exact upper and lower edge exists. In one-dimensional space the requirement of distributivity of operations of addition and multiplication is satisfied. Thus, due to the Stone's theorem, in a one-dimensional physical space the set of the observer's states can always be represented as various subsets of elementary events occurring in it. The information corresponding to the point "A" in this space is the information state of the "Laplace's demon", which "knows" about all the events which occurred in the lower light cone of the point "A". The fact of principal importance is that for the ordering of states of the "Laplace's demon" we do not use this set of events, but do it by means of the algorithmic theory of information.

Concerning the three-dimensional physical space, the exact upper and lower edge for a random pair of events in it does not exist. Nevertheless, we assume that even in this case the proof of the generalized analog of the Stone's theorem is possible, as well as the representation of the partially ordered set of the observer's states (Laplace's demon) in the form of various subsets of elementary events ordered in the three-dimensional space-time. It is quite possible that it will require the use of quantum-mechanical formalism (in particular, the Pauli matrixes), while the elementary events will possess quantum properties (absence of localization, entangled states, etc.).

However, the obtained space is neither vector, nor continuous, nor symmetrical. In order to allow the obtained results to "open" the fundamental laws of motion in the information space, let us use the "clues" from physics: B. Green writes [6]: "One of the universal lessons of the last century is that the known laws of physics are in correspondence with the principals of symmetry… For instance, why would the reference system of one observer be more preferable than another observer's system? On the contrary, from the point of view of fundamental laws of the universe it seems much more natural to interpret all reference systems equally". Considering the information space, we can also say: "Why would one language of description (programming) be more preferable than another? On the contrary, it seems much more natural to interpret all languages equally".

At the first stage the simplest types of symmetries can be taken into account for the set of methods of description of information objects. They are identical to the introduction of a subset of inertial reference systems in physics. They appear to be sufficient for the construction of an information analog of the special theory of relativity. However, at the same time, the description of the "trajectories of motion" in all "non-inertial" languages of description requires the introduction of additional external forces and fields.

Expansion of the symmetry of languages in such a way that it would include all relative "non-inertial" motions will likely result in the occurrence of an information analog of the general theory of relativity. Further expansion of the types of used symmetries is connected with the discreteness of representation of information objects, and, apparently, will require the use of the quantum-mechanical approach.

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