White-collar Employment, New Technologies, and Relative Wages in a R&D-based Growth Model*

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July 19, 2004

Abstract

This paper develops a R&D-based growth model to examine the relationship between technological change, growth, and the demand for skill-intensive, analytical activities (e.g., product development, quality-control, and design of advertising campaigns). Results are consistent with evidence on rising employment shares of skilled, white-collar workers and increases in the skill premium in the US or UK. Moreover, accounting for a simultaneous decrease in overhead labor requirements (e.g., administrative staff), the analysis suggests that recent technology shifts have no systematic impact on firm sizes and on the economy’s rate of growth. This sheds some light into the “Solow-productivity paradox”. Finally, the analysis suggests that a higher effectiveness of advertising may increase growth and welfare, even if advertising activity is purely wasteful from a social point of view.

Key words: Advertising; Analytical Skills; Information Technology; Skill Premium; R&D-based Growth; White-collar Employment.

JEL classification: O31, O33, J21, J31.

*Acknowledgements: I am grateful to an anonymous referee for very valuable comments. This paper has also benefited from discussions with Henning Bohn, Josef Falkinger, Volker Meier, and seminar participants at the University of Zurich and the conference on “Dynamics, Economic Growth, and International Trade” (DEGIT VIII) 2003, Helsinki.

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1 Introduction

There is now a broad consensus that in the 1980s and 1990s the structure of labor demand has shifted in favor of skilled workers, and that this shift can largely be attributed to technological advances. However, understanding the interplay between new technologies and the structure of labor demand requires to shed light into the decision of firms how to allocate workers with different skills to different tasks. For instance, contrary to earlier notions of “skill-biased technological change” (SBTC), computer users per se do not seem to have gained more from computerization than workers using a pencil (DiNardo and Pischke, 1997). In fact, administrative workers, who nowadays use computers more intensively than most other group, seem to be clear losers of the recent technological revolution (e.g. Berman, Bound and Grichilis, 1994). Such evidence demonstrates that the mere observation that shifts in the labor demand structure are related to the emergence of new information and communication technologies (ITC) is not a very useful hypothesis.

This paper attempts to draw a more differentiated picture on the relationship between technological change, growth and the structure of employment by hypothesizing that computerization has favored skill-intensive, analytical activities like product development, quality-control, design of customer services, and promotion of products by advertising campaigns. These tasks require much analytical thinking and are based on efficient flows of information about markets and customers. Creation and analysis of such information are favored by new ITC. New ITC allow marketing managers to assemble, store and analyze customer data like demographics and purchase habits (“data mining”). In turn, ‘data warehouses’ enable firms to design and keep track of marketing campaigns and to target consumers more effectively than by mass-media advertising (e.g. Bresnahan, 1999; Shapiro and Varian, 1999). Moreover, enhanced possibilities to do research on consumers’ preferences, the emergence of computer-aided design and more efficient interactions between design, production and marketing help firms to improve the quality of products.
I develop a non-scale endogenous growth model which allows for both types of demand-enhancing tasks, R&D, performed in-house in order to improve the quality of goods, and advertising, which is viewed as promotional activity.\(^1\) Firms can freely enter the economy but have to cover the costs for non-production labor from subsequent profits under monopolistic competition. This is because R&D and advertising activities are incurred prior to product market competition, and thus give rise to endogenous sunk cost. Moreover, firms have to incur fixed overhead labor requirements in terms of both skilled or unskilled workers, which may be interpreted to include administrative staff.

First, it is argued that an increase in the effectiveness of skill-intensive, quality-improving (R&D) or promotional activities fosters a reallocation of skilled workers from production-related activities towards these analytical, demand-enhancing tasks. This shift in the employment structure is consistent with evidence provided by the empirical literature on SBTC, which shows a clear upward trend in the share of skilled, white-collar workers like managers and professionals (e.g., Berman, Bound and Grichilis, 1994; Berman, Bound and Machin, 1998; Machin and van Reenen, 1998; see also Falkinger and Grossmann, 2003). Moreover, the wage-bill of firms for skilled, white-collar workers unambiguously increases, endogenously implying higher sunk costs of firms. Consequently, the number of firms declines, and thus, firm sizes increase. In contrast, an equiproportionate decrease in the fixed overhead labor requirements (e.g., a decrease in administrative overhead costs) neither affects aggregate employment in demand-enhancing tasks nor relative wages, and, as usual, raises the number of firms. Interestingly, however, a decrease in fixed costs reduces the rate of growth. Although somewhat surprising at the first glance, this result is consistent with the empirical evidence that larger firms conduct more R&D (e.g. Cohen and Levin, 1989; Cohen and Klepper, 1996).

In sum, the analysis suggests that a rising effectiveness of demand-enhancing tasks, together with a decline in overhead requirements, leads to both shifts in the employment

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\(^1\)See Grossmann (2003) for an extensive discussion of this modelling approach and the mechanisms which arise from introducing promotional activity in a quality-ladder growth model featuring in-house R&D. In that paper, however, I do neither allow for heterogenous agents nor for wage inequality.
structure towards skilled, white-collar workers and rising skill premia, however, without affecting firm sizes and growth in a systematic way. Hence, the proposed theory is consistent with two further empirical regularities. First, it sheds some light into the so-called “Solow productivity paradox” ("...you can see the computer age everywhere but in the productivity statistics", Solow, 1987). Empirical evidence suggests that the administrative staff in firms has declined significantly. Although this may raise welfare due to an increase in product variety, a positive relationship between firm size and R&D activity gives rise to a growth-retarding effect of declining overhead costs. This serves as a counteracting force to a positive impact of an increase in the effectiveness of R&D on growth. As a result, productivity may not increase.\(^2\) Second, despite the merger wave in the 1980s and 1990s, there is no clear evidence on rising firm sizes (e.g. Pryor, 2001).

The model also points to an interesting interplay between R&D and advertising incentives of firms. Besides an increase in the effectiveness of R&D, also a more effective advertising technology, even if intensifying a wasteful competition among firms, may raise both growth and welfare. This is because higher sunk costs incurred for advertising are associated with an increase in firm sizes. Firm size, in turn, is positively related to innovation activity in the proposed framework. As a counteracting effect, however, due to the assumption that promotional activity is skill-intensive, an increase in the demand for advertising raises the skill premium. This gives a disincentive for firms to hire researchers, leaving the relationship between advertising incentives and innovation activity ambiguous. If an increase in the effectiveness of advertising raises growth, however, it may also raise welfare, despite a decline in product variety which is triggered by higher advertising outlays.

\(^2\)For instance, Falkinger and Grossmann (2003, Tab. 1) show that the U.S. employment share of workers in administrative occupations in the manufacturing sector has declined from 12.8 percent in 1983 to 9.1 percent in 2000. In producer services (banking, insurance, real estate, legal services etc.), the decline in the employment share of administrators was even more pronounced, having decreased from 35.1 to 19.5 percent during that time period.

\(^3\)However, evidence suggests that it has done so in the second half of the 1990s (contrary to the evidence in the 1980s and early 1990s) - at least in the US (e.g. Stiroh, 2002).
Most theoretical studies on SBTC do not account for differences in tasks performed by production-related and non-production labor. (See Acemoglu, 2002, for a comprehensive review of this literature.) An exception is an interesting, related model by Nahuis and Smulders (2002), who argue that an increase in the supply of skilled workers fosters a shift towards a more knowledge-intensive production process, requiring more non-production workers. As a result, if the intertemporal return from innovations can be appropriated by firms to a sufficiently large degree, the skill premium permanently rises. In contrast to their study, the present analysis focusses on technological changes regarding analytical tasks, rather than on skill supply. Other growth models focussing on labor reallocation towards innovation activity and shifts in relative wages are developed by Grossmann (2000) and Thesmar and Thoenig (2001). However, their contributions differ from the present one in that they do not consider in-house R&D and do not allow for advertising. Moreover, Grossmann (2002) examines an ideal variety model which shows that standard notions of skill-biased process innovations are typically not consistent with a rise in skill premia when allowing for skill-intensive, quality-improving tasks.

Other related literature, although not addressing wage inequality and advertising, is concerned with the relationship between R&D and concentration. Smulders and van de Klundert (1995) first formalized the empirical finding that big firms can spread the cost of R&D over a larger volume of sales in a growth model (see also Peretto, 1998, 1999),4 which plays a crucial role for the impact of a change in fixed costs and advertising on growth in the present paper.

Finally, alternative explanations of the Solow productivity paradox refer to measurement problems of both output (particularly in service industries) and quality-improvements of goods, as well as costs of adjustment to new technologies (for a comprehensive discussion, see Triplett, 1999). Regarding adjustment costs, Bas and Nahuis (2002) argue that skilled labor is temporarily withdrawn from production in

4Cohen and Klepper (1996) provide a simple IO model which rests on the cost-spreading hypothesis, and present empirical evidence in favor of it.
order to accumulate knowledge after the introduction of a new general purpose tech-
nology, resulting in both higher wage inequality and (for some time) lower productivity
growth.

The plan of the paper is as follows. Section 2 presents the model. The equilibrium
analysis is provided in section 3. Section 4 summarizes the main hypotheses on a shift
in the demand for analytical skills. Section 5 examines the impact of an improvement in
the advertising technology on welfare. The last section concludes. Proofs are relegated
to an appendix.

2 The Model

Consider an economy which is populated by \( L \) individuals with infinite lifetimes, each
supplying one unit of labor in each period \( t = 0, 1, 2, \ldots \) (i.e., there is no population
growth). There is a segmented and perfectly competitive labor market with two types
of labor, \( L^S \) skilled and \( L^U \) unskilled workers (i.e., \( L = L^S + L^U \)), which differ in
their analytical ability. There exists a (positive) representative consumer (who chooses
aggregate market demand when endowed with aggregate resources) with intertemporal
utility function

\[
U = \sum_{t=0}^{\infty} \rho^t \ln C_t,
\]

where \( C_t \) is a consumption index, which is given by

\[
C_t = \left( \int_{0}^{n_t} (q_t(i)x_t(i))^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}},
\]

where \( \sigma > 1 \). \( x_t(i) \) denotes the quantity of good \( i \in N_t \equiv [0, n_t] \) consumed in period \( t \), whereas
\( q_t(i) \) will be referred to as its perceived quality. Each firm produces exactly one variety
of the horizontally differentiated product in monopolistic competition. The measure
\( n_t \) is referred to as the “number of firms” at date \( t \) and is endogenously determined.

Firms have a constant-returns to scale production technology. To keep the analysis
as simple as possible, assume that the production function $F$ for differentiated goods is of the Cobb-Douglas type:\footnote{This specification is inconsequential for the main results of the paper (as discussed below), and allows to derive all results analytically. The crucial assumption is the linear-homogeneity of the production function.}

$$x_t(i) = F(l^S_t(i), l^U_t(i)) = a \left( l^S_t(i) \right)^\alpha \left( l^U_t(i) \right)^{1-\alpha},$$

(3)

$a > 0$, $0 < \alpha < 1$, where $l^S_t(i)$ and $l^U_t(i)$ denote skilled and unskilled production-related labor in firm $i \in \mathcal{N}_t$, respectively, $t \geq 0$.

There are two types of demand-enhancing, non-production activities: quality-improving tasks performed in-house ("R&D") and promotional tasks ("advertising"). These activities are more skill-intensive than production-related tasks. For simplicity, suppose they can only be performed by skilled labor. For instance, R&D may be interpreted as product innovations or improvement of customer services whereas advertising may be viewed as framing of product characteristics in accordance with consumers’ desires. Both (in-house) R&D labor investments and advertising costs have to be incurred one period in advance of production, i.e., are sunk in the production period.\footnote{The assumption that current R&D spending of a firm is effective in the subsequent (production) period follows Young (1998).} Formally, perceived product quality $q_t(i)$ of variety $i$ in any period $t > 0$ evolves according to

$$q_t(i) = \begin{cases} \bar{S}_{t-1} g(l^{R}_{t-1}(i)) h(l^{A}_{t-1}(i)/\bar{l}^{A}_{t-1}) & \text{if } g(l^{R}_{t-1}(i)) \geq 1, \\ \bar{S}_{t-1} h(l^{A}_{t-1}(i)/\bar{l}^{A}_{t-1}) & \text{otherwise}, \end{cases}$$

(4)

where $l^{R}_{t-1}(i)$ and $l^{A}_{t-1}(i)$ denote the amount of R&D and advertising labor of firm $i \in \mathcal{N}_t$ employed in $t - 1$, respectively, and

$$\bar{l}^{A}_{t-1} = \frac{1}{n_t} \int_0^{n_t} l^{A}_{t-1}(i) di,$$

(5)

$t \geq 1$, is the average amount of advertising labor of firms producing consumption goods
in $t$. Both $g(\cdot)$ and $h(\cdot)$ are increasing functions. Note that, if all firms allocate the same amount of labor to advertising (i.e., $l_{t-1}^A(i) = \bar{l}_{t-1}^A > 0$ for all $i$), no firm gains compared to a situation without advertising. That is, engaging in promotional activity is a form of wasteful competition.\footnote{The modelling strategy that only the relative advertising effort matters for a firm’s success in affecting perceived quality is similar to the game-theoretic literature on “contest success functions” (Skaperdas, 1996), applied here to a general equilibrium model with monopolistically competitive firms.}

$$\tilde{S}_{t-1} = \tilde{S}_{t-2} \frac{1}{n_{t-1}} \int_0^{n_{t-1}} g(l_{t-2}^R(i)) di$$ (6)

reflects an intertemporal knowledge spillover effect from previous investments of firms in R&D. Regarding intellectual property rights, (4) and (6) imply that innovations are proprietary knowledge for one period only. Moreover, (4) and (6) borrow from Young (1998) in modelling “equivalent innovations”. That is, if all firms invest the same amount in R&D at date $t-2$, i.e., if $l_{t-2}^R(i) = l_{t-2}^R$ for all $i$, we have $\tilde{S}_{t-1} = \tilde{S}_{t-2} g(l_{t-2}^R)$, according to (6). That is, the number of firms conducting research at date $t-2$, $n_{t-1}$, does not affect research capabilities of firms in the subsequent period. Intuitively, this means that firms come up with similar solutions to similar problems at the same time.

In the model of Young (1998), this assumption eliminates the empirically problematic feature of many endogenous growth models that the economy’s growth rate depends on population size (“scale effect”).\footnote{See Jones (1995) and Young (1998) for more discussion.} As will become apparent in section 3.3, in the present model the steady-state growth rate depends on the relative supply of skilled labor, $L^S/L^U$, but not on population size $L$.

The number of firms $n_0$ in the initial period is historically given. Moreover, for simplicity, assume $q_0(i) = \tilde{S}_0 > 0$, $i \in \mathcal{N}_0$, for the firms’ initial product quality. Also specify $g(l^R) = (l^R)^\kappa$, $h(l^A/\bar{l}^A) = (l^A/\bar{l}^A)^\eta$, (7)

$\kappa > 0$, $\eta > 0$. The parameters $\kappa$ and $\eta$ are referred to as “effectiveness of R&D” and
“effectiveness of advertising”, respectively.

There is free entry of firms into the economy, with a large number of ex ante identical potential entrants. At all times, firms have to incur fixed labor requirements \( f^S(\geq 0) \) and \( f^U > 0 \) in terms of skilled and unskilled labor, respectively, prior to production. These fixed labor requirements may be interpreted as including administrative staff (e.g., concerning tasks like supervising, billing, auditing etc.). Although, in general, administrative tasks are not literally independent of output, it is plausible to assume that bureaucracy costs have a fixed component, which is the crucial element in the present context. As outlined in the introduction, recent developments suggest that technological change has reduced these overhead costs. In \( t - 1 \), firms which produce final output in period \( t \) issue bonds or shares in a perfect asset market in order to finance fixed (labor) costs as well as non-production labor costs for R&D and advertising.

3 General Equilibrium

Let us choose unskilled labor as numeraire and denote the (relative) wage rate of skilled labor in period \( t \) by \( \omega_t \). The representative consumer’s budget constraint in period \( t \geq 0 \) then reads\(^9\)

\[
A_{t+1} = (1 + r_t)A_t + \omega_t L^S + L^U - E_t,
\]

(8)

where \( A_t \) denotes the value of asset holdings in \( t \), \( E_t \) is consumption expenditure and \( r_t \) is the (endogenous) interest rate between \( t - 1 \) and \( t \). Utility maximization implies that consumption spending evolves according to Euler equation

\[
E_t = (1 + r_t)\rho E_{t-1},
\]

(9)

\(^9\)Initial income from asset holdings \((1 + r_0)A_0\) is exogenously given for consumers. In addition to budget constraint (8), the representative consumer also has to observe both a standard transversality condition, which is given by \( \lim_{T \to \infty} A_{T+1} / \prod_{t=1}^{T} (1 + r_t) = 0 \), and non-negativity constraints, \( E_t \geq 0 \), \( A_{t+1} \geq 0 \), \( t \geq 0 \).
$t > 0$. Moreover, the demand function for good $i$ in period $t$ is given by
\begin{equation}
x^D_t(i) = q_t(i)^{\sigma-1}E_t \left( \frac{p_t(i)}{P_t} \right)^{-\sigma},
\end{equation}
where $p_t(i)$ is the price of good $i$ in $t$. The price index
\begin{equation}
P_t \equiv \left( \int_0^n \left( \frac{p_t(i)}{q_t(i)} \right)^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}
\end{equation}
is defined such that the CES-index $C_t$, given by (2), equals real consumption expenditure in period $t$, i.e., $C_t = E_t/P_t$.

Cost minimization implies that the (relative) wage rate for skilled labor fulfills
\begin{equation}
\omega_t = \frac{\alpha}{1 - \alpha} \frac{l^U_t(i)}{l^S_t(i)},
\end{equation}
and marginal production cost are given by
\begin{equation}
c_t = \frac{(\omega_t)^\alpha}{a\alpha^\alpha(1 - \alpha)^{1-\alpha}},
\end{equation}
according to production technology (3). Profits of firm $i$ in period $t$ are given by $\pi_t(i) = (p_t(i) - c_t)x^D_t(i)$. Thus, using (10), output prices by the monopolistically competitive firms are set according to the well-known formula
\begin{equation}
p_t(i) = p_t = \frac{\sigma}{\sigma - 1}c_t
\end{equation}
for all $i \in \mathcal{N}_t$ and $t \geq 0$ (Dixit and Stiglitz, 1977).

Non-production labor costs of any firm $i \in \mathcal{N}_t$ equal $\omega_{t-1} \left( l^R_{t-1}(i) + l^A_{t-1}(i) + f^S \right) + f^U$. (Recall that unskilled labor is numeraire.) To avoid only mildly interesting case distinctions, let us focus the analysis on the case $g(l^R_{t-1}(i)) \geq 1$. Thus, at time $t - 1,
the firm value $V_{t-1}(i)$ of firm $i \in \mathcal{N}_t$ is given by
\begin{equation}
V_{t-1}(i) := \max_{l_{t-1}^R(i), l_{t-1}^A(i)} \left\{ \frac{P_t - c_t}{1 + r_t} x_t^D(i) - \omega_{t-1} \left( \frac{l_{t-1}^R(i)}{l_{t-1}^A(i)} + l_{t-1}^A(i) + f^S \right) - f^U \right\}
\tag{15}
\end{equation}
\begin{align*}
\text{s.t. } x_t^D(i) &= \left[ \tilde{S}_{t-1} g(l_{t-1}^R(i)) h \left( \frac{l_{t-1}^A(i)}{l_{t-1}^R(i)} \right) \right]^{\sigma-1} E_t \left( \frac{p_t}{P_t} \right)^{-\sigma},
\end{align*}
according to (4), (10) and (14). Note that, since each single firm has measure zero (i.e., there are no strategic interactions among firms), $P_t$, $E_t$ and $l_{t-1}^A$ are taken as given in the optimization problem of firms. Using (7), it is easy to show that under
\begin{equation}
(\kappa + \eta)(\sigma - 1) < 1,
\tag{A1}
\end{equation}
product demand $x_t^D(i)$ as stated in (15), and thus each firm’s objective function at date $t - 1$, is strictly concave as function of $(l_{t-1}^R(i), l_{t-1}^A(i))$. As all potential entrants are identical, the analysis focusses on symmetric equilibria; that is, for all $i \in \mathcal{N}_t$, we have $l_{t-1}^R(i) = \bar{l}_{t-1}^R$, $l_{t-1}^A(i) = \bar{l}_{t-1}^A$, $V_{t-1}(i) = V_{t-1}$, $t \geq 1$, and $l_{t}^S(i) = \bar{l}_{t}^S$, $l_{t}^U(i) = \bar{l}_{t}^U$, $x_t^D(i) = x_t^D$, $t \geq 0$.

Recall that we focus on $g(l_{t-1}^R) \geq 1$, and thus, under (7), $l_{t-1}^R \geq 1$. Moreover, as will become apparent, $l_{t-1}^A > 0$ under (A1). Thus, the first-order conditions of maximization program (15) can be written as equality:
\begin{align*}
\frac{p_t - c_t}{1 + r_t} x_t^D (\sigma - 1) \frac{g'(l_{t-1}^R)}{g(l_{t-1}^R)} &= \omega_{t-1}, 
\frac{p_t - c_t}{1 + r_t} x_t^D (\sigma - 1) \frac{h'(l_{t-1}^A)}{h(1)} \frac{1}{l_{t-1}^A} &= \omega_{t-1}.
\tag{16}
\tag{17}
\end{align*}
Conditions (16) and (17) simply state that marginal benefits and marginal costs of R&D and advertising employment, respectively, are equalized. Combining (16) with (17), and using (7), the following first result is implied.

**Lemma 1.** For any $t \geq 1$, we have $l_{t-1}^A / l_{t-1}^R = \eta / \kappa$ in symmetric equilibrium.
Hence, the ratio of advertising employment to R&D employment is time-invariant. It decreases with the effectiveness of R&D, $\kappa$, and increases with the effectiveness of advertising, $\eta$.

In symmetric equilibrium, the following conditions must hold under free entry (FE), clearing of goods markets (GM) as well as clearing of labor markets for skilled and unskilled workers, (LMS) and (LMU), respectively.\(^{10}\)

\[
V_{t-1} = \frac{p_t - c_t}{1 + r_t} x^D_t - \omega_{t-1} \left( l^R_{t-1} + l^A_{t-1} + f^S \right) - f^U = 0, \ t \geq 1; \quad \text{(FE)}
\]
\[
x^D_t = a \left( l^S_t \right) \alpha \left( l^U_t \right)^{1-\alpha}, \ t \geq 0; \quad \text{(GM)}
\]
\[
L^S = n_{t-1} l^S_{t-1} + n_t \left( l^R_{t-1} + l^A_{t-1} + f^S \right), \ t \geq 1; \quad \text{(LMS)}
\]
\[
L^U = n_{t-1} l^U_{t-1} + n_t f^U, \ t \geq 1. \quad \text{(LMU)}
\]

We are now ready to study the general equilibrium implications of both changes in incentives of firms to incur costs for R&D and advertising, reflected by $\kappa$ and $\eta$, respectively, and shifts in fixed labor requirements $f^S, f^U$. (Equilibrium values are denoted by superscript (*) throughout the paper.)

### 3.1 Relative Wages

Which kind of technological changes are consistent with a rise in wage inequality, as observed particularly in Anglo-American economies throughout the 1980s and most of the 1990s? The following result provides an answer. (All results are proven in appendix.)

**Proposition 1.** (Wage inequality.) Under (A1), in equilibrium, the (relative) wage rate of skilled labor is time-invariant, i.e., $\omega_t = \omega^*$ for all $t \geq 0$, where $\omega^*$ increases with $\kappa$ and $\eta$, and is homogenous of degree zero as function of $(f^S, f^U)$.

An increase in $\kappa$ or $\eta$ makes skilled labor more effective in analytical non-production work.

\(^{10}\) According to Walras’ law, these conditions imply that also the asset market clears.
tasks. This gives firms incentives to reallocate skilled labor from production to non-production activities, leaving skilled labor a scarcer resource. Consequently, the relative wage rate $\omega^*$ increases, in line with the empirical evidence for the US and UK in the 1980s and (at least) the early 1990s.

Moreover, an equiproportionate change in $f^S$ and $f^U$ (leaving $f^S/f^U$ unchanged) has no impact on $\omega^*$. In fact, one can show that this property holds for any constant-returns to scale technology for the production of final goods, represented by function $F$. One way to understand this is that equilibrium production labor inputs per firm, $l^S$ and $l^U$, are inversely related to the number of firms, which - not surprisingly - is increasing in both $f^S$ and $f^U$ (as will become apparent below). Thus, technological change which reduces skilled and unskilled labor requirements in the same proportion (i.e., an equiproportionate decrease in $f^S$ and $f^U$) cannot explain a change in wage inequality.

The absence of transitional dynamics in the model is not confined to the relative wage, $\omega$. Formally, the underlying reason for this property lies in the linear spillover effect in the evolution of perceived quality (4), which leads to a time-invariant interest rate. In sum, we obtain the following.

**Corollary 1.** Under (A1), the equilibrium interest rate immediately jumps to a steady state level, with $r_t = (1 - \rho)/\rho$ for all $t \geq 1$. Moreover, in equilibrium, $E$, $p$, $n$, $l^R$, $l^A$, $l^S$ and $l^U$ are time-invariant from period 1 onwards, whereas $l^S_0 \neq l^S_t$ and $l^U_0 \neq l^U_t$ whenever $n_0 \neq n_t$, $t \geq 1$.

Let us denote $E_{t-1} = E^*$, $p_{t-1} = p^*$, $n_t = n^*$, $l^R_{t-1} = l^{R*}$, $l^A_{t-1} = l^{A*}$, $l^S_t = l^{S*}$, and $l^U_t = l^{U*}$ for equilibrium values in $t \geq 1$; moreover, denote $l^S_0 = l^{S*}_0$ and $l^U_0 = l^{U*}_0$ regarding the equilibrium at period 0.

### 3.2 Number and Size of Firms

Recall that there are two types of sunk costs in the model: endogenous costs for R&D and advertising labor as well as the exogenous overhead costs $f^S$ and $f^U$ in terms of
skilled and unskilled labor, respectively (e.g. for administration). As usual, sunk costs
give rise to economies of scale which determine the equilibrium number of firms, $n^*$,
and thus, firm sizes, $L/n^*$, under free entry. This is reflected in the following result.

**Proposition 2.** (Firm size.) Under (A1), for any $t \geq 1$, steady state firm size,
$L/n^*$, increases with the effectiveness of R&D or advertising, $\kappa$ or $\eta$, respectively;
moreover, $L/n^*$ is linear homogenous as function of $(f^S, f^U)$.

As usual, an increase in exogenous fixed costs, reflected by labor requirements
$f^S$ and $f^U$, reduces the number of firms $n^*$, and thus, raises firm sizes $L/n^*$. More
interestingly, since an increase in R&D or advertising incentives, $\kappa$ or $\eta$, raise demand
for skilled, white-collar workers, sunk costs (i.e., the wage-bill for white-collar workers)
endogenously rise, in turn raising firm sizes.

### 3.3 Innovations and Growth

What are the determinants of innovation activity and economic growth? Let $\vartheta_t$ denote
the growth rate of real consumption $C_t = E_t/P_t$; i.e., define $\vartheta_t \equiv (C_t - C_{t-1})/C_{t-1}$.
We obtain the following result.

**Proposition 3.** (R&D, advertising and growth). Under (A1), for any $t > 1$:

(i) The economy’s growth rate is given by $\vartheta_t = \vartheta^* = g(l^{Rs}) - 1 = (l^{Rs})^\kappa - 1$;

(ii) both $l^{Rs}$ and $\vartheta^*$ are increasing in $\kappa$, whereas the impact of $\eta$ on $l^{Rs}$ and $\vartheta^*$ is
ambiguous;

(iii) $l^{Rs}$ is linear-homogenous as function of $(f^S, f^U)$, i.e., an equiproporionate
increase in $f^S$ and $f^U$ raises $\vartheta^*$;

(iv) $l^{Rs}$ is homogenous of degree zero as function of $(L^S, L^U)$, i.e., an equiproporationate
increase in $L^S$ and $L^U$ does not affect $\vartheta^*$; both $l^{Rs}$ and $\vartheta^*$ are increasing in $L^S/L^U$;

(v) $l^{As}$ is increasing in $\eta$ and linear-homogenous as function of $(f^S, f^U)$, whereas
the impact of $\kappa$ on $l^{As}$ is ambiguous.

13
Not surprisingly, according to part (i) of Proposition 3, the steady-state growth rate $\vartheta^*$ rises when $l^{Rs}$ rises. The intuition of part (ii) is as follows. A shift in the effectiveness of R&D, $\kappa$, raises $l^{Rs}$ by increasing the marginal benefit of innovation activity, which is given by the left-hand side of first-order condition (16). An increase in the effectiveness of advertising, $\eta$, has two counteracting effects on R&D labor per firm, $l^{Rs}$. On the hand, an increase in $\eta$ endogenously raises the firms’ sunk costs for advertising. This positively affects innovation activity per firm, since the marginal benefit to invest in R&D increases when firms become larger (i.e., when $x^D$ increases, all other things equal).\footnote{This mechanism, which relies on a positive relationship between innovation activity and firm size, has been extensively studied e.g. in Smulders and van de Klundert (1995) and Peretto (1998, 1999). For empirical support, see e.g. Cohen and Levin (1989) and Cohen and Klepper (1996).} Second, however, it raises the wage rate $\omega^*$ for skilled labor, according to Proposition 2. This means that researchers become more expensive, implying a disincentive to invest in R&D. A priori, it is not clear which effect dominates. Regarding part (iii), again, due to a positive relationship between firm size and innovation incentives (and the constant-returns-to-scale production technology), technological change which induces an equiproportionate decrease in $f^S$ and $f^U$ lowers the economy’s rate of growth. This result is related to an absence of a “scale effect” regarding the rate of growth in the model, which is established in part (iv). That is, $\vartheta^*$ is independent on population size $L = L^S + L^U$ of the economy, although being positively affected by relative skill supply $L^S/L^U$. If, to the contrary, the number of firms, and thus the scale of the economy, would matter for growth, then a decrease in exogenous fixed costs may spur growth by raising the number of firms. Finally, comparative-static results regarding advertising employment per firm, $l^{A*}$, are analogous to the results regarding $l^{Rs}$.

### 3.4 Aggregate White-collar Employment of Skilled Labor

As argued in the introduction, the observed increase in the employment share of skilled, white-collar workers, particularly in managerial and professional occupations,
has been taken as evidence for the hypothesis of SBTC. Under the interpretation of $f^S$ as skilled, administrative staff, the aggregate equilibrium employment of skilled, white-collar workers is given by $\Gamma^* \equiv n^* (l^{R*} + l^{A*} + f^S)$.

**Proposition 4.** (Skilled, white-collar employment). Under (A1), for any $t \geq 0$, total skilled, white-collar employment, $\Gamma^*$, is increasing in both $\kappa$ and $\eta$, and homogeneous of degree zero as function of $f^S$, $f^U$.

Proposition 4 shows that an increase in the demand for skilled labor, when triggered by a higher $\kappa$ or $\eta$, is not only reflected by a higher skill premium, $\omega^*$, and higher R&D and advertising activity per firm, $l^{R*}$ and $l^{A*}$, but also in higher aggregate white-collar employment of skilled labor, $\Gamma^*$. That is, despite the negative impact of an increase in R&D and advertising incentives on the number of firms (Proposition 2), there is a reallocation towards skilled, white collar employment in the aggregate. In contrast, an equiproportionate decline in administrative staff, $f^S$ and $f^U$, has no impact on $\Gamma^*$.

In sum, an empirical prediction of our analysis, which explicitly distinguishes between production-related and non-production activities of skilled labor, is that computerization has increased the demand for analytical skills in non-production activities (which are reflected in sunk costs). The analysis is thus capable to shed light into the sources of observed shifts in the labor demand structure. This is further discussed in the next section.

### 4 The Shift in Demand for Analytical Skills

Theorem 1 summarizes the main hypotheses suggested by the preceding analysis.

**Theorem 1.** Under (A1).

(a) An increase in the effectiveness of R&D or advertising ($\kappa$ or $\eta$, respectively) together with an equiproportionate decrease in overhead labor requirements ($f^S$ and $f^U$)

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12 Defining $\Gamma_t \equiv n_{t+1} (l_t^{R} + l_t^{A} + f^S)$, $t \geq 0$ (recall that $n_{t+1}$ is the number of firms which produce consumption goods in $t+1$, requiring investments at date $t$) yields $\Gamma_t = \Gamma^*$ for all $t \geq 0$, according to Corollary 1.
raises relative wages, \( \omega^* \), and the employment share of skilled, white-collar workers, \( \Gamma^* \), without affecting firm sizes, \( L/n^* \), and the growth rate, \( \vartheta^* \), in a systematic way.

(b) Fiercer wasteful competition for customers by advertising (i.e., an increase in \( \eta \)) may have a positive impact on growth.

Empirical evidence shows that computerization has not favored computer users per se, but, in contrast, has led to a substitution of employees in administrative occupations by computerized routines. In the present model, this is reflected by a decrease in \( f^S \) and \( f^U \). At the same time, computerization has enabled firms to create and analyze large customer databases (e.g., Bresnahan, 1999). This has opened up new possibilities for market research, contributing to a better understanding of consumer behavior. In turn, firms were enabled to find out how to frame product characteristics more effectively in their advertising campaigns, to keep track of advertising campaigns, and to target potential customers more directly. These developments are reflected by an increase in \( \eta \). Maybe even more important, more effective market research, the emergence of computer-aided design, and more efficient information flows between production, development and marketing units have helped to improve the quality of products and services, which is reflected by an increase in \( \kappa \). Under these hypotheses, part (a) of Theorem 1 suggests that computerization has raised both the skill premium and total employment in demand-enhancing activities by raising demand for analytical skills (since both \( \omega^* \) and \( \Gamma^* \) increase with \( \kappa \) and \( \eta \), whereas being unrelated to equiproportionate decreases in \( f^S \) and \( f^U \)). However, computerization is neither systematically related to firm sizes nor to economic growth.

According to part (b) of Theorem 1, higher spending on promotional activities (triggered by an increase in \( \eta \)), although modelled as being completely wasteful from a social point of view, may raise the economy’s rate of growth. This result is due to the (empirically well-supported) properties of the model that higher sunk costs (e.g., for advertising) raise firm size, and, in turn, larger firms conduct more R&D. However, there is a counteracting effect since advertising incentives are positively related to the
wage rate of skilled labor, in turn leaving researchers more expensive.

As a remark, the assumption that innovation activity affects product quality, \( q_t \), rather than productivity is made merely for the sake of concreteness. Alternatively, treating R&D as *productivity-enhancing* activity (e.g., reorganization of production, development of management techniques, creation of an internal human capital stock), rather than being related to the quality of goods, may formally lead to exactly identical results. To see this, suppose that (alternatively to (4)) perceived quality can be affected by advertising only, i.e., let \( q_t(i) = h(l_{t-1}^A(i)/\bar{l}_{t-1}^A) \). Moreover, let the production function (similar to (3)) be given by \( x_t(i) = A_t(l_t^S(i))^{\alpha} (l_t^U(i))^{1-\alpha} \), where total factor productivity \( A_t \) is given by \( A_t = \bar{S}_{t-1}g(l_{t-1}^R(i)) \) if \( g(l_{t-1}^R(i)) \geq 1 \) (and \( A_t = \bar{S}_{t-1} \) otherwise). Suppose that everything else remains the same. It is straightforward to show that, under this modification, all formal results remain *exactly* valid. Only the type of innovation activity has changed from affecting \( q_t \) to affecting \( A_t \). For instance, similar to the creation of customer databases which raise the potential to improve the quality of goods, computerization has also allowed to assemble data on the internal organization of a firm (Bresnahan, 1999). In turn, analytical skills are needed to draw conclusions from these extended possibilities to improve production processes (and thus, to raise \( A_t \)). Again, this raises the demand for skilled, white-collar workers. According to part (a) of Theorem 1, however, the overall impact of technological change on productivity growth is generally ambiguous, consistent with the “Solow-productivity paradox”.

## 5 Welfare Effects of Advertising

According to part (b) of Theorem 1, advertising incentives, measured by \( \eta \), may raise growth. A natural question to ask is whether an increase in \( \eta \) may also raise intertemporal welfare in equilibrium, \( U^* \), despite lowering product variety unambiguously.

This section examines the relative importance of declining product variety versus potentially growth-enhancing effects of a more effective advertising technology for wel-
In order to show that an increase in $\eta$ may increase welfare, an example suffices. For gaining some insight into the plausibility of such an outcome, however, the only simplification made here is to specify $f^S = 0$. In this case, the following can be stated.

**Proposition 5.** (Advertising and welfare.) Suppose $f^S = 0$ and (A1) hold. Then the impact of an increase in $\eta$ on welfare $U^*$ may generally be positive or negative. It is negative if $\eta$ is sufficiently high.

As apparent from the proof in the appendix, both possibilities, a positive or negative relationship between the effectiveness of advertising, $\eta$, and welfare, $U^*$, occur under plausible parameter values. The result that an increase in $\eta$ may raise welfare can be led back to the feature of the model that firm size and the marginal return to R&D are positively related, and may be more than a theoretical peculiarity. It is ultimately a consequence of complex interactions between several market imperfections in the model: imperfect goods markets, a positive intertemporal externality of R&D, and a negative (static) externality of advertising.

### 6 Conclusion

This paper has developed a R&D-based growth model to examine the relationship between relative wages of skilled labor, the structure of employment in production-related and analytical tasks, firm sizes, economic growth, and welfare. It has been argued that the emergence of new ITC has favored skill-intensive, analytical activities, which are related to sunk costs of firms. For the sake of concreteness, the analysis has focussed on demand-enhancing activities like product development, quality-control, and

\[ \text{Comparing the market allocation to the socially optimal allocation, and providing an analysis of optimal tax/subsidy policies are the main focus of the companion paper to this article (Grossmann, 2003), in which labor is homogenous. Such an analysis, however, is beyond the scope of the present paper.}

\[ \text{In Grossmann (2003), a potentially positive welfare effect of improvements in the advertising technology cannot occur when an interior solution to the social planning optimum exists. One can check, however, that this is not the case in the present context.} \]
design of customer services, and advertising. As outlined, however, the results equally apply for a study of productivity-enhancing activities.

It has been shown that higher incentives to invest in R&D or advertising lead to a reallocation of skilled labor towards these analytical tasks, in turn, raising relative wages of skilled labor. In contrast to the standard literature on SBTC, which does not distinguish between production-related and non-production tasks, results are not only consistent with a rising skill premium in (fairly) flexible labor markets like the US and the UK, but also with rising employment shares in managerial and professional activities. Empirical evidence also suggests that the administrative staff, although fairly intensive computer users, has been downsized considerably. Accounting for this decrease in overhead labor requirements, in addition to hypothesizing a higher effectiveness of analytical tasks, the analysis suggests that recent technology shifts have neither a systematic impact on firm sizes nor on the economy’s rate of growth. The latter is consistent with the “Solow productivity paradox” which refers to the puzzle that computerization did not seem to have helped boosting productivity in a significant way at least until the mid 1990s.

Finally, the interplay between innovation activity and promotional activity has been examined. In particular, it has been shown that higher advertising incentives, although intensifying a wasteful competition in the model, may lead to faster growth. This result rests on the sunk cost nature of advertising spending together with the property that larger firms have higher incentives to innovate, all other things equal. In addition, and even more surprising, this mechanism gives rise to a potentially positive impact of a technology-related increase in advertising incentives on welfare.

Appendix

Proof of Proposition 1. Note that \( h(1) = 1 \), according to (7). Thus, (4) implies that \( q_t(i) = q_t = \tilde{S}_{l-1}g(l_t^{R}) \) for all \( i \) in symmetric equilibrium. Hence, together with \( p_t(i) = p_t \) from (14), equations (10) and (11) imply that demand for each differentiated
good is given by
\[ \frac{x_t^D}{n_t p_t} = \frac{E_t}{n_t}. \]  
(18)

Substituting Euler equation (9) into (18), and using (14), leads to
\[ \frac{p_t - c_t}{1 + r_t} \frac{x_t^D}{n_t} = \frac{\rho E_{t-1}}{\sigma n_t}. \]  
(19)

Next, note that substituting (19) into free entry condition (FE) implies
\[ \frac{\rho E_{t-1}}{\sigma n_t} = \omega_{t-1} \left( t_{t-1}^R + t_{t-1}^A + f^S \right) + f^U. \]  
(20)

Moreover, substituting (7) and (19) into first-order condition (16) yields
\[ \frac{\rho E_{t-1}}{\sigma n_t} (\sigma - 1) \kappa = \omega_{t-1} t_{t-1}^R. \]  
(21)

Now substitute (20) into (21) and use \( t_{t-1}^A = \eta_t^R / \kappa \) from Lemma 1 to obtain
\[ \omega_{t-1} t_{t-1}^R = \frac{\kappa (\sigma - 1) \left( \omega_{t-1} f^S + f^U \right)}{1 - (\kappa + \eta)(\sigma - 1)}. \]  
(22)

Substitution of (22) into (21) then leads to
\[ \frac{E_{t-1}}{n_t} = \sigma \left( \omega_{t-1} f^S + f^U \right) \frac{1}{\rho [1 - (\kappa + \eta)(\sigma - 1)]}. \]  
(23)

\( t \geq 1 \). Moreover, combining equilibrium condition (GM) with (18), using (12)-(14), and rearranging terms yields
\[ n_{t-1} t_{t-1}^U = E_{t-1} \frac{(\sigma - 1)(1 - \alpha)}{\sigma}. \]  
(24)
for any \( t \geq 1 \). Substituting (24) into labor market clearing condition (LMU) and using (23) then leads to

\[
\frac{1}{n_t} = \frac{1}{L^U} \left[ \frac{(\sigma - 1)(1 - \alpha) \left( \omega_{t-1} f^S + f^U \right)}{\rho \left[ 1 - (\kappa + \eta)(\sigma - 1) \right]} + f^U \right].
\]  
(25)

This gives a first relationship between \( n_t \) and \( \omega_{t-1} \). Next, note that (12) implies \( l_{t-1}^L = \omega_{t-1} l_{t-1}^S (1 - \alpha) / \alpha \), \( t \geq 1 \). Substituting this into (24) and rearranging terms yields

\[
n_{t-1} l_t^S = \frac{E_t}{\omega_{t-1}} - \frac{(\sigma - 1)(1 - \alpha)}{\rho \left[ 1 - (\kappa + \eta)(\sigma - 1) \right]} \left( f^S + f^U \right) + f^S.
\]  
(26)

Substituting (26) and \( l_t^A = l_{t-1}^R \eta / \kappa \) from Lemma 1 into labor market clearing condition (LMS), and substituting both (22) and (23) into the resulting expression, eventually leads to a second relationship between \( n_t \) and \( \omega_{t-1} \):

\[
\frac{1}{n_t} = \frac{1}{L^S} \left[ \frac{(\sigma - 1)(\alpha + \rho(\kappa + \eta)) \left( \omega_{t-1} f^S + f^U \right)}{\rho \left[ 1 - (\kappa + \eta)(\sigma - 1) \right]} + f^S \right].
\]  
(27)

Combining (25) and (27) then proves that the relative wage is time-invariant in equilibrium, i.e., \( \omega_{t-1} = \omega^* \) for all \( t \geq 1 \), where \( \omega^* \) is implicitly given by

\[
0 = \left( \sigma - 1 \right) \left[ \alpha + \rho(\kappa + \eta) \left( \frac{f^S}{f^U} + \frac{1}{\omega^*} \right) + \frac{f^S}{f^U} \rho \left[ 1 - (\kappa + \eta)(\sigma - 1) \right] \right] - \frac{L^S}{L^U} \left( \sigma - 1 \right) (1 - \alpha) \left( \omega^* \frac{f^S}{f^U} + 1 \right) + \rho \left[ 1 - (\kappa + \eta)(\sigma - 1) \right].
\]  
(28)

Thus, applying the implicit function theorem,

\[
\frac{\partial \omega^*}{\partial \kappa} = \frac{\rho \left( \frac{L^S}{L^U} + \frac{1}{\omega^*} \right)}{\frac{\alpha + \rho(\kappa + \eta)}{\omega^*^2} + (1 - \alpha) \frac{L^S f^S}{L^U f^U}} > 0,
\]  
(29)

and, similarly, \( \partial \omega^*/\partial \eta > 0 \). Finally, (28) implies that \( \omega^* \) is homogenous of degree zero as function of \((f^S, f^U)\). This concludes the proof. 

**Proof of Corollary 1.** First, according to (25) [or (27)], \( \omega_{t-1} = \omega^* \) implies that the
number of firms is time-invariant, \( n_t = n^* \), \( t \geq 1 \). Thus, according to (23), \( E_{t-1} = E^* \) for all \( t \geq 1 \). Combining this with (9) confirms the expression for the interest rate, \( r_t \). Also note that
\[
p_t = p^* = \frac{\sigma (\omega^*)^\alpha}{(\sigma - 1) \alpha \alpha^\alpha (1 - \alpha)^{1-\alpha}}
\]
for all \( t \geq 0 \), according to (13), (14), and \( \omega_t = \omega^* \), i.e., output prices are time-invariant in equilibrium. Moreover, \( \omega_{t-1} = \omega^*, \ t \geq 1 \), implies that \( l^R \) and \( l^A \) are time-invariant, according to (22) and Lemma 1, respectively. Moreover, \( n_t = n^*, \ E_{t-1} = E^* \) for all \( t \geq 1 \) imply that \( l^U_t \), and, together with \( \omega_t = \omega^* \), also \( l^S_t \) is constant from period 1 onwards, according to (24) and (26), respectively. (24) and (26) also imply that \( l^U_0 \neq l^U_t \) and \( l^S_0 \neq l^S_t \), respectively, if \( n_0 \neq n_t, \ t \geq 1 \). This concludes the proof. ■

Proof of Proposition 2. The result immediately follows from (25) [or (27)] and Proposition 1. ■

Proof of Proposition 3. To prove part (i), first, confirm by using the expression for the price index (11) together with \( E_t = E^*, \ p_t = p^* \) and \( n_{t+1} = n^*, \ t \geq 0 \), that \( \vartheta_t \) equals the growth rate of perceived quality \( q_t \). Since, using \( h(1) = 1 \),
\[
q_t = \tilde{S}_{t-1} g(l^{R*}) h(1) = \tilde{S}_0 g(l^{R*})^t,
\]
in symmetric equilibrium, according to (4) and (6), \( q_t \) and thus \( \vartheta_t \) grow with rate \( g(l^{R*}) - 1 \). This confirms part (i). Note from part (i) that the effects on \( l^{R*} \) immediately imply the effects on \( \vartheta^* \), which are thus not stated separately in the following proofs of parts (ii)-(iv). To prove part (ii), note that (22) and \( \omega_{t-1} = \omega^* \) imply
\[
l^{R*} = \frac{\kappa (\sigma - 1) \left( f^S + \frac{f^U}{\omega^*} \right)}{1 - (\kappa + \eta) (\sigma - 1)}.
\]
Using both (28) and (29), it is tedious but straightforward to confirm that \( \partial l^{R*} / \partial \kappa > 0 \). In contrast, \( \partial l^{R*} / \partial \eta >, =, < 0 \) is possible, which confirms part (ii). Part (iii), which states that \( l^{R*} \) is linear-homogenous as function of \( (f^S, f^U) \), follows from (31) and the
fact that $\omega^*$ is homogenous of degree zero as function of $\left( f^S, f^U \right)$. Part (iv) follows from (31) and the fact that $\omega^*$ is homogenous of degree zero as function of $\left( L^S, L^U \right)$ and is decreasing in $L^S/L^U$, according to (28). By analogy, the results regarding $l^{A*} = l^{R*} \eta/\kappa$ (recall Lemma 1) follow from the results regarding $l^{R*}$, which confirms part (v). This concludes the proof. ■

**Proof of Proposition 4.** Using (31) and $l^{A*} = l^{R*} \eta/\kappa$ from Lemma 1, one obtains

$$l^{R*} + l^{A*} + f^S = \frac{(\kappa + \eta)(\sigma - 1) f^U + f^S}{1 - (\kappa + \eta)(\sigma - 1)}. \quad (32)$$

Moreover, using $\omega_{t-1} = \omega^*$, (25) implies

$$n^* = \frac{L^U \rho [1 - (\kappa + \eta)(\sigma - 1)]}{(\sigma - 1)(1 - \alpha) (\omega^* f^S + f^U) + f^U \rho [1 - (\kappa + \eta)(\sigma - 1)]}. \quad (33)$$

Recall that $\Gamma^* = n^* (l^{R*} + l^{A*} + f^S)$. According to (32) and (33), and making use of (28), we thus get

$$\Gamma^* = \frac{(\kappa + \eta)(\sigma - 1) \omega^* + f^S}{(\sigma - 1) [\alpha + \rho(\kappa + \eta)] \left( \frac{f^S}{f^U} + \frac{1}{\omega^*} \right) + f^S \rho [1 - (\kappa + \eta)(\sigma - 1)]}. \quad (34)$$

Homogeneity of degree zero of $\Gamma^*$ as function of $\left( f^S, f^U \right)$ immediately follows from (34) and Proposition 1. Using (34) and $\partial \omega^*/\partial \kappa > 0$ or $\partial \omega^*/\partial \eta > 0$, respectively, the impact of an increase in $\kappa$ or $\eta$ on $\Gamma^*$ is straightforward but tedious to confirm. This concludes the proof. ■

**Proof of Proposition 5.** First, note that $f^S = 0$ implies

$$\omega^* = \frac{L^U}{L^S} \frac{(\sigma - 1) [\alpha + \rho(\kappa + \eta)]}{(\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1)]}, \quad (35)$$

$$n^* = \frac{L^U}{f^U} \frac{\rho [1 - (\kappa + \eta)(\sigma - 1)]}{(\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1)]}, \quad (36)$$
and

\[ t^{R*} = \frac{\kappa f L^S}{L^U} \frac{(\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1)]}{[\alpha + \rho (\kappa + \eta)][1 - (\kappa + \eta)(\sigma - 1)]}, \]

(37)

according to (28), (31) and (33). Moreover, combining (2)-(6) and observing Corollary 1, one obtains that, for all \( t \geq 1 \), \( C_t = a S_0 g(t^{R*}) l^n_\star (l^S_\star)^{\alpha - 1} (l^U_\star)^{1 - \alpha} \) in equilibrium (recall \( h(1) = 1 \)). Thus, using (7) and (12),

\[ \ln C_t = t \kappa \ln t^{R*} + \frac{\sigma}{\sigma - 1} \ln n^* + \ln l^{U*} - \alpha \ln \omega^* + \text{const.}, \]  

(38)

\( t \geq 1 \). Analogously, observing \( q_0(i) = \bar{S}_0 \) for all \( i \in \mathcal{N}_0 \), \( C_0 = a S_0 (n_0) \hat{\sigma} (l^S_0)^{\alpha} (l^U_0)^{1 - \alpha} \) in equilibrium, and thus,

\[ \ln C_0 = \ln l^{U*}_0 - \alpha \ln \omega^* + \text{const.} \]  

(39)

according to (2) and (12). Now, substituting (38) and (39) into (1), and making use of both \( \sum_{t=0}^{\infty} \rho^t = 1/(1 - \rho) \) and \( \sum_{t=1}^{\infty} \rho^t t = \rho/(1 - \rho)^2 \), leads to

\[ U^* = \ln l^{U*}_0 + \frac{1}{1 - \rho} \left( \rho \ln l^{U*} - \alpha \ln \omega^* + \frac{\rho \sigma}{\sigma - 1} \ln n^* + \frac{\rho \kappa}{1 - \rho} \ln t^{R*} \right) + \text{const.} \]  

(40)

Next, setting \( f^S = 0 \) in (23), combining the resulting expression with (36), and observing Corollary 1, yields

\[ E^* = \frac{\sigma L^U}{(\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1)]}. \]

(41)

Combining \( E_{t-1} = E^* \) as given in (41) with (24) leads to

\[ l^{U*}_0 = \frac{1}{n_0} \frac{(\sigma - 1)(1 - \alpha) L^U}{(\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1)]}. \]

(42)

Moreover, according to (24), (36) and (41),

\[ l^{U*} = \frac{(\sigma - 1)(1 - \alpha) f^U}{\rho [1 - (\kappa + \eta)(\sigma - 1)]}. \]

(43)
Now, substituting (35), (36), (37), (42) and (43) into (40), and manipulating the resulting expression, one can show that

\[ U^* = \text{const.} + \frac{\rho \kappa}{1 - \rho} \ln \kappa + \frac{\rho [1 - \kappa (\sigma - 1) - \rho]}{(\sigma - 1)(1 - \rho)^2} \left( \ln [1 - (\kappa + \eta) \sigma - 1] - \ln f^U \right) - (44) \]

\[ \frac{\alpha (1 - \rho)}{1 - \rho} + \rho \kappa \ln [\alpha + \rho (\kappa + \eta)] - \frac{(1 - \rho)(\sigma - 1)(1 - \alpha) + \rho [1 - \kappa (\sigma - 1) - \rho]}{(\sigma - 1)(1 - \rho)^2} \times \]

\[ \ln ((\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1))] \]

From this, it is easy to derive that

\[ \frac{\partial U^*}{\partial \eta} = -\frac{\rho}{(1 - \rho)^2} \left( \frac{\alpha (1 - \rho) + \rho \kappa}{\alpha + \rho (\kappa + \eta)} + \frac{\sigma - 1}{(1 - \alpha)} \frac{[\eta (1 - \rho) - \rho \kappa]}{[1 - (\kappa + \eta) \sigma - 1][(\sigma - 1)(1 - \alpha) + \rho [1 - (\kappa + \eta)(\sigma - 1)]]} \right) \]  \hspace{1cm} (45)

Thus, \( \partial U^*/\partial \eta < 0 \) if, e.g., \( \eta (1 - \rho) \geq \rho \kappa \). In contrast, if, for instance, \( \eta = 0.1, \kappa = 0.2, \sigma = 4 \), (i.e., \( 1 - (\kappa + \eta)(\sigma - 1) = 0.1 \), thus fulfilling assumption (A1)), \( \rho = 0.9 \) and \( \alpha = 0.5 \), then \( \partial U^*/\partial \eta > 0 \), according to (45). This confirms the result.

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