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Regularities in stock markets

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From the stock markets of six countries with high GDP, we study the stock indices, *S&P 500 (NYSE, USA)*, *SSE Composite (SSE, China)*, *Nikkei (TSE, Japan)*, *DAX (FSE, Germany)*, *FTSE 100 (LSE, Britain)* and *NIFTY (NSE, India)*. The daily mean growth of the stock values is exponential. The daily price fluctuations about the mean growth are Gaussian, but with a finite asymptotic convergence. The growth of the monthly average of stock values is statistically self-similar to their daily growth. The monthly fluctuations of the price follow a Wiener process, with a decline of the volatility. The mean growth of the daily volume of trade is exponential. These observations are globally applicable and underline regularities across global stock markets.

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I. INTRODUCTION

The main concerns of a stock market analyst are the percentage growth rate of stock values and the susceptibility of stock prices to fluctuations. Speculation in the stock market is encouraged by the optimism that markets grow over time scales that are long enough to average out the impact of adverse fluctuations in stock values. Ever since Bachelier [1] proposed his theory of speculation, many formal approaches have afforded continuous refinement in the analysis of financial data (see [2, 3] and all references therein for a historical development of the subject). One object of such researches is to discern universal features in stock data [4], whose statistical properties do not vary overmuch, irrespective of the market. Financial data of world markets do exhibit certain features that appear to be broadly common, and as such, markets attain some predictability. Understanding these familiar patterns inspires greater confidence in the speculative activity.

Our work here has a similar purpose. We study stock indices from the stock markets of six countries that are in the top ten of global ranking of GDP. They are USA, China, Japan, Germany, Britain and India. Our chosen stock indices are *S&P 500 (NYSE, USA)* [5], *SSE Composite (SSE, China)* [6], *Nikkei (TSE, Japan)* [7], *DAX (FSE, Germany)* [8], *FTSE 100 (LSE, Britain)* [9] and *NIFTY (NSE, India)* [10]. We examine stock indices rather than individual stocks, because stock indices cover multiple companies across a wide range of sectors. Thus, stock indices reflect the overall condition of markets more comprehensively than would an individual stock [4, 11, 12]. The health of a market can be gauged fairly through the daily price movement of its stock indices, the fluctuations they undergo, and the daily volume of stocks traded. For these variables, we arrive at well-restricted ranges of numerical values across the markets we have studied. We see that the mean daily growth of the value of stock indices is exponential (Sec. II). The daily fluctuations of a stock index about its exponentially growing mean value has a Gaussian distribution, but with a finite asymptotic convergence for large fluctuations (Sec. III). The growth of the average monthly value of a stock index is statistically self-similar to its daily growth (Sec. IV). The monthly variance of the price follows a Wiener process [2, 13], with the volatility reducing progressively in time (Sec. IV). The mean growth of the number of daily transaction of stocks is exponential (Sec. V). We present our results graphically using the data of *NIFTY (NSE, India)* [10], from January, 1997 to April, 2019. Similar results from five other stock markets [5–9] are summarized in Table I. The conclusions derived from our analysis of financial data of stock markets of six high-GDP countries are globally valid.

II. THE DAILY GROWTH OF MEAN STOCK PRICES

The forward relative change of a stock price, S, in a finite time interval, Δt, is given by [2, 13]

$$\frac{\Delta S}{S} = a\,\Delta t + b\,\Delta W, \tag{1}$$

in which ΔW expresses a Wiener process [2, 13] about a background exponential growth of S, implied by the first term on the right hand side of Eq. (1). Under an idealized volatility-free condition we set $b = 0$ in Eq. (1), and then integrate it in continuous time to get a steady compounded growth of S. The integral solution of S is exponential in time, $S = S_0 \exp(at)$. We, however, express this volatility-free equation slightly differently as $\Delta(\ln S) = a\,\Delta t$, and fit it with the data of the stock index, *NIFTY (NSE, India)*. The fit is shown in Fig.1, which is a linear-log plot of the movement of the daily average price of *NIFTY (NSE, India)* over more than two decades, from January, 1997 to April, 2019. The mean growth of $\ln S$, fitted by the least-squares method, is linear in Fig.1, demonstrating thereby that the daily mean growth of S occurs exponentially.

The daily movement of five other stock indices, i.e. *S&P 500 (NYSE, USA)*, *SSE Composite (SSE, China)*, *Nikkei (TSE, Japan)*, *DAX (FSE, Germany)* and *FTSE 100 (LSE, Britain)*, display the same graphical trend as in Fig.1, over time scales of two decades. In all cases, the mean relative growth rate of S is a, whose values are provided in Table I. Globally, a is confined within a range of 0.01% to 0.05% per day, which seems to be surprisingly narrow and precise.

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TABLE I: Summary of the analysis of financial data taken from six stock markets, whose names are listed in the first column. Regularities across global markets are discernible through the numerical values of the parameters catalogued below.

Stock Index	a (% per day)	μ	σ	m (per month)	w (per month)	ν (% per day)
S&P 500 (NYSE, USA)	0.03	0.032	1.203	0.006	-2.78×10^{-6}	–
SSE Composite (SSE, China)	0.04	0.101	2.805	0.009	-3.09×10^{-5}	0.12
Nikkei (TSE, Japan)	0.01	0.027	1.494	0.003	-1.02×10^{-6}	0.53
DAX (FSE, Germany)	0.03	0.025	1.466	0.006	-4.42×10^{-6}	0.02
FTSE (LSE, Britain)	0.01	0.011	1.165	0.002	-1.78×10^{-6}	0.003
NIFTY (NSE, India)	0.05	0.057	1.495	0.010	-3.41×10^{-6}	0.04

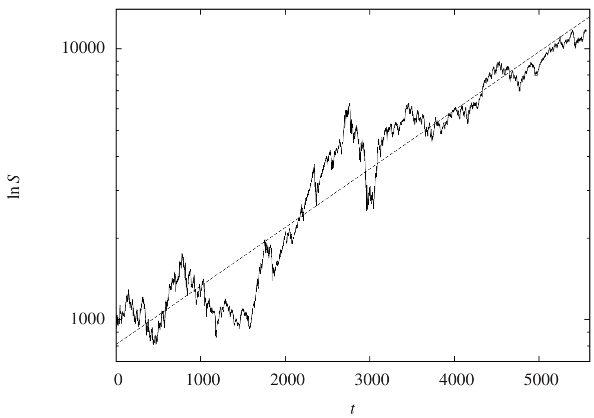

FIG. 1: The daily mean growth of the average price of the stock index, *NIFTY (NSE, India)*. The linear-log plot is modelled by Eq. (1). Here time, t, is measured in days, over more than two decades. The straight line in this linear-log plot is fitted by the least-squares method, and indicates that the mean growth of S is exponential. In Eq. (1), with $b = 0$, the mean relative growth rate of stock values is a. For this plot, $a = 0.05$% per day. Values of a for all the stock indices are provided in the second column of Table I.

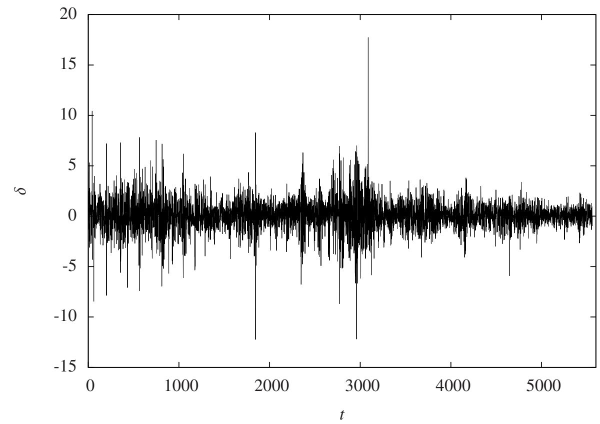

FIG. 2: The time series of the daily percentage fluctuation of prices in the stock index, *NIFTY (NSE, India)*. As in Fig.1, the time, t, is measured in days. The daily percentage fluctuation of prices is quantified by δ, which, over two decades, has an equal distribution of positive and negative values about $\delta = 0$. Most of the fluctuations are small, but a very few fluctuations have large values in both extremes. Taken together, all these features are captured by Eq. (2), which accommodates the possibility of the rare but large fluctuations.

III. GAUSSIAN FLUCTUATIONS IN STOCK PRICES

By setting $b = 0$ in Eq. (1), we have so far ignored the fluctuations about the mean exponential growth of a stock price, S. In reality, however, as we can see very clearly in Fig.1, stock prices fluctuate noticeably about the mean exponential growth (represented by the straight line in Fig.1). To quantify the fluctuations, we define a new variable, δ, which is the daily percentage change that a stock index undergoes, with respect to the value of the previous day. Positive fluctuations are given by $\delta > 0$, and negative fluctuations by $\delta < 0$. The time series of such fluctuations, for the stock index, *NIFTY (NSE, India)*, is plotted in Fig.2, which shows that positive and negative fluctuations are balanced about $\delta = 0$. Most of the fluctuations are close to $\delta = 0$, with a very few cast far away. The unnormalized frequency distribution of these fluctuations has a Gaussian profile in Fig.3, but with an asymptotic convergence towards unity. To model all these aspects, we put forward a Gaussian function, with unity added to it, as

$$f(\delta) = 1 + f_0 \exp\left[-\frac{(\delta - \mu)^2}{2\sigma^2}\right], \tag{2}$$

in which the mean of the frequency distribution, $\mu = \langle\delta\rangle$, and the standard deviation, $\sigma = \sqrt{\langle\delta^2\rangle - \mu^2}$. The values of both μ and σ, calibrated with the data of our chosen six stock indices, are listed in Table I. With the frequency distribution clustering around $\delta = 0$ in Fig.3, we expect μ to have a small value. Supporting this expectation, Table I shows us that μ is zero up to the first place of decimal for all the stock indices, except *SSE Composite (SSE, China)*. Likewise, the first significant figure of σ is unity, except for *SSE Composite (SSE, China)*.

The theory of unbiased and random fluctuation of stock prices, described by a continuous Gaussian function, was introduced in financial markets by Bachelier [1]. The randomness of prices is a response to a ceaseless stream of unpredictable and uncorrelated information that enters the market continuously. While this process plays out on the microscopic scale of individual prices, its macroscopic imprint is manifested through a similar randomness in the market indices. Fluctuations in financial markets are, therefore, a collective macro-outcome of a diversity of microscopic causes that are independent of one another. On the whole, the randomness agrees theoretically with the Gaussian function, but only if the time scale of the fluctuations is long enough [2, 11, 12],

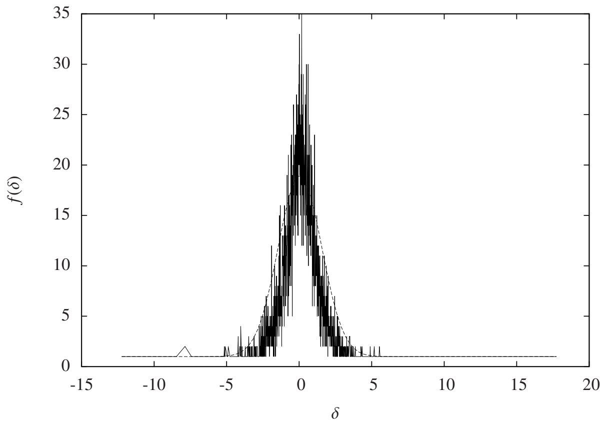

FIG. 3: The unnormalized frequency distribution of the daily percentage fluctuation of prices in the stock index, *NIFTY (NSE, India)*. The distribution appears Gaussian, and is centred around a mean value, $\mu = 0.057$, with a standard deviation, $\sigma = 1.495$. For large fluctuations, the asymptotic convergence is $f(\delta) \longrightarrow 1$, as Eq. (2) confirms. Values of μ and σ for all the stock indices are to be found, respectively, in the third and fourth columns of Table I.

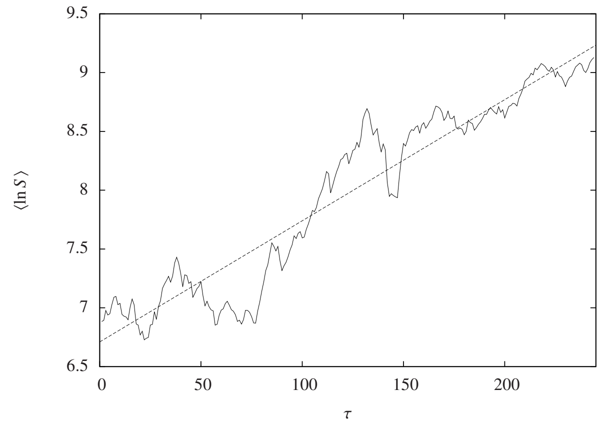

FIG. 4: The growth of the monthly average of $\ln S$ for *NIFTY (NSE, India)*, as opposed to its daily growth in Fig.1. The time, τ, is scaled in months, and spans more than two decades. The straight line, showing the mean growth, is fitted by the least-squares method, and its slope is $m = 0.01$ per month. Values of m for all the stock indices are in the fifth column of Table I. This plot is statistically self-similar to the plot in Fig.1, despite differing widely in time scales.

such as the scale of a day in our study. On shorter time scales, however, market indices exhibit scaling properties [14], leading to an inverse cubic law of stock value fluctuations [4, 15]. Such fluctuations converge more slowly towards an asymptotic limit than the Gaussian function does. Slow convergence is the hallmark of power laws, but we can mimic a slow convergence even with a Gaussian function, by adding a constant of unity to the right hand side of Eq. (2). This reconciles a continuous Gaussian function with a frequency distribution that is discrete. The consequence of the discreteness is that for large fluctuations, the unnormalized frequency distribution will asymptotically converge to unity. When $\delta \longrightarrow \infty$, the form of Eq. (2), despite being Gaussian in essence, theoretically ensures the convergence, $f(\delta) \longrightarrow 1$.

IV. THE WIENER PROCESS AND VOLATILITY

Looking back at Eq. (1), we consider first the basic nature of a Wiener process. If a variable, W, undergoes a Wiener process, then its change, ΔW, in a discrete interval of time, Δt, is $\Delta W = \epsilon\sqrt{\Delta t}$ [13]. Here ϵ has a standardized normal distribution with a zero mean and a standard deviation of unity [13]. For different short intervals, Δt, ΔW will have independent values. Over a time interval, T, the mean, $\langle \Delta W \rangle = 0$, and the standard deviation, $\sqrt{\langle (\Delta W)^2 \rangle} \sim T^{1/2}$ [13].

Stock price variations follow a generalized Wiener process [13], viewed also as a geometric Brownian motion [2, 3, 13]. The generalized Wiener process has a non-zero mean that drifts linearly in time [13], as the first term on the right hand side of Eq. (1) suggests. About this linear mean drift, there is a stochastic component, given by the second term on the right hand side of Eq. (1), now to be read as $\Delta W = \epsilon\sqrt{\Delta t}$. On a daily time scale, t, the mean growth is depicted by the

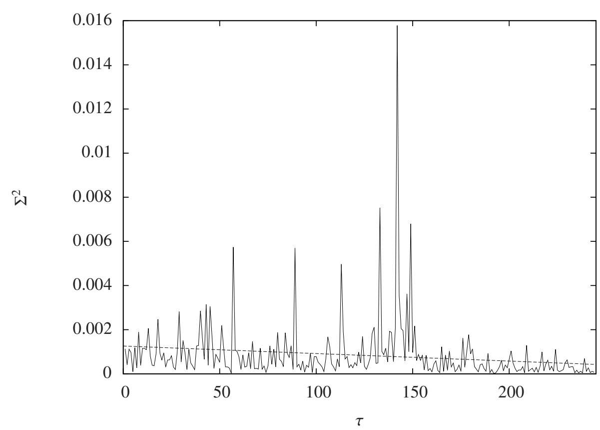

FIG. 5: The Wiener variance about the monthly average of $\ln S$ for *NIFTY (NSE, India)* decreases with time, τ (in months). The straight line, fitted by the least-squares method, traces the mean decline, with a slope of $w = -3.41 \times 10^{-6}$ per month. With $w < 0$, volatility also reduces with time. Values of w for all the stock indices are in the sixth column of Table I. The tallest spike in this plot coincides with the global economic recession around the year, 2008, and all stock markets show a similar spike about the same year.

straight line in Fig.1, while the stochastic fluctuations (related to the volatility of the stock value [13]) are seen in the jagged features about the straight line. Likewise, all through January, 1997 to April, 2019, on a monthly time scale, which we write as τ to distinguish it from t, the straight line in Fig.4 shows the mean growth of the monthly average of the natural logarithm of the price of the stock index, *NIFTY (NSE, India)*. About the mean linear growth of $\langle \ln S \rangle$, whose slope is m in Fig.4, the jaggedness shows the usual stochastic behaviour — the volatility. Notwithstanding the noticeable difference in the time scales, i.e. a day for Fig.1 and a month for Fig.4, there

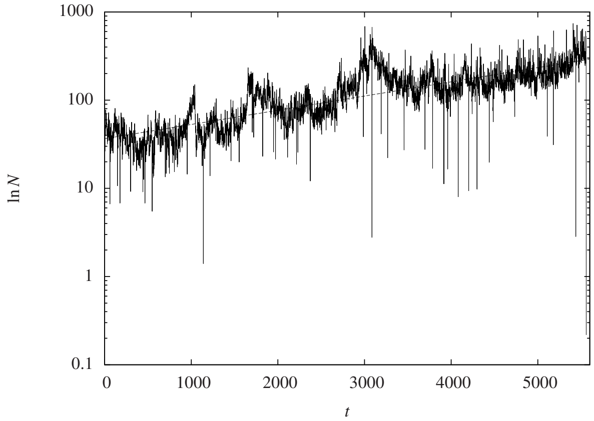

FIG. 6: The growth of the daily trade volume of the *NIFTY (NSE, India)* index. The number of daily transactions, N, is scaled by 10^6, and the time, t, is scaled in days. The straight line in this linear-log plot, fitted by the least-squares method, implies an exponential mean growth of N. The slope of the straight line, $\nu = 0.04\%$ per day, gives the mean relative growth rate of the daily volume of trade. Values of ν for all the stock indices are in the last column of Table I.

is a statistical self-similarity between the two plots. This is a generic characteristic of all the stock indices that we have studied. Consistent with this observation, in Table I we find that across all stock indices, there is a high correlation coefficient of 0.987 between a and m.

From the *NIFTY (NSE, India)* index, besides estimating $\langle \ln S \rangle$ on a monthly time scale, we also estimate the standard deviation of $\ln S$ over the same month. The standard deviation, Σ, is related to the volatility of the stock price [13]. This volatility is clearly revealed by the fluctuations about the linearly growing mean of $\langle \ln S \rangle$ in Fig.4. We also see in Fig.4 that the fluctuations reduce progressively as both τ and S increase. Hence, the volatility of the stock price subsides with the passage of time. We can understand this phenomenon for a Wiener process by recasting Eq. (1) as $\Delta(\ln S) = a\Delta t + b\epsilon\sqrt{\Delta t}$, on whose right hand side, the first term stands for the mean relative growth of S and the second term stands for the fluctuations about the mean growth. In an interval of time, Δt, the mean growth varies linearly with Δt, while the fluctuations vary as $(\Delta t)^{1/2}$. Thus, on large time scales, the growth of $\ln S$ is dominated by the linear mean growth, against which the fluctuations become much weakened. Scaling the time in months, this is exactly what we see in Fig.4. Further to this point of view, Fig.5 demonstrates how the variance of the monthly prices, Σ^2, generally decreases with time, τ. A quantitative measure of this decline is found in the slope, w, of the straight line, $\Sigma^2 = w\tau$, fitted with the data in Fig.5 by the least-squares method. With $w < 0$ for the *NIFTY (NSE, India)* index, as well as for all the other stock indices in Table I, stability against volatility is seen to be a general tendency of mature stock markets.

In passing, we note that Fig.5 has a prominent spike in the monthly variance. This spike corresponds to the economic recession that disrupted global markets around the year, 2008. This disruption is universally captured by a very tall spike in the monthly variance of all the stock indices we have studied.

V. THE DAILY VOLUME OF TRADE

A reliable yardstick of the health of a stock market is the number of daily transactions in the market. The linear-log plot in Fig.6 shows an exponential mean growth of the trade volume at 0.04% per day for the *NIFTY (NSE, India)* index. Other global markets display the same behaviour, going by the relative growth rate of their daily transactions in Table I.

VI. CONCLUSIONS

The indices of the stock markets of six countries with high GDP show that markets experience long-term growth of stock values and the amount of trade. Mature markets tend to be stable against volatility. These qualitative views are quantitatively favoured by the numerical values we present in Table I. The closeness of the values of each parameter in Table I satisfies our quest for regularities in stock markets.

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