Reply to Simon Gilson on Diminishing Utility

Victor Aguilar www.axiomaticeconomics.com

Abstract

In a series of e-mails, Simon Gilson has proposed a modification to Axiom 2ii, regarding diminishing utility. Must it really be negative monotonic?

In Axiomatic Theory of Economics, I claim that marginal (diminishing) utility, $u(s)$, is such that:

- 2i) It is independent of first-unit demand.
- 2ii) It is negative monotonic; that is, $u'(s) < 0$.
- 2iii) The integral of $u(s)$ from zero to infinity is finite.

Axiom 2i is necessary because a separate axiom defines what is required of first-unit demand. As long as a $u(s)$ function meets conditions 2ii and 2iii, it can describe how value diminishes regardless of how valuable the first unit is; these are proportions, not absolute drops in value.

Axiom 2iii is necessary because my theory would fail without it. Without convergence, the demand for everything at any price would be infinite, which is clearly not the case. This makes me unique; all economists have stated in a vague sort of way that utility diminishes, but none have ever put this more restrictive condition on it.

For instance, diminishing utility can be defined by e^{-s} but not by $\frac{1}{s}$. Other economists would look at the graphs of these two functions, observe that they look similar and assume that they are both viable. Their own theories also fail if utility diminishes too slowly, but that is something they never consider because they never bothered to clearly state their axioms.

Axiom 2ii is necessary only to thwart critics who are purposefully trying to find a counter-example – no matter how contrived – so they can say that my theory does not work in some cases. A big bump far out in the tail would throw a monkey wrench into the mathematics. But it does not actually matter if utility increases as long as it eventually diminishes with resolve. Potato chips are the classic example – nobody can eat just one. Of course, potato chips are not sold individually but by the bag and people *do* typically buy just one bag at a time. But there may be some nontrivial examples of this phenomenon. This is allowable provided that at some point we can state with assurance that there will be no further upticks.

Simon Gilson writes, "Perhaps it does not matter, and you merely need to specify that there exists some s^* for which all $s > s^*$ imply that $u'(s) < 0$." True. Theorem 2 (*Axiomatic Theory of Economics*, p. 107) states, "There exists an N such that, for all $n >$ N, $u(n) < \frac{1}{n}$." The proof of this theorem is the only one that invokes Axiom 2ii and it remains true even under the weaker axiom proposed by Gilson.

Theorem 2 was a bit of an afterthought. When I was a senior in college (1992) I showed a preliminary draft of my book to one of my math professors, Dr. Kumjian, and he pointed out that I was tacitly assuming $u(n) < \frac{1}{n}$. I thought this was clear because $u(s)$ converges and $\frac{1}{n}$ does not but, at his insistence, I proved Theorem 2, which necessitated Axiom 2ii, though this axiom is stronger than necessary. Indeed, I could have just had Axiom 2ii read, "There exists an N such that, for all $n >$ N, $u(n) < \frac{1}{n}$." This is the weakest version of 2ii possible, though less intuitive than the version that I will adopt: The tail is negative monotonic; that is, there exist an s^* such that, for all $s > s^*$, $u'(s) < 0$.

Note that weaker means less restrictive and is a good thing because it gives the theory wider applicability; people unfamiliar with deductive logic often see the word "weaker" and assume that it is something bad. The weakest version allows upticks in the tail that are prohibited by stating that the tail is negative monotonic, though only very slight upticks that keep $u(s) < \frac{1}{s}$.

The fact is that the shape of the $u(s)$ function does not much matter. When I wrote the software simulation, I tested it with a variety of utility functions and could see no visible difference in the demand distribution. When it was still a QBasic program, users could modify the $u(s)$ function. But when it became a Java Applet, I made diminishing utility an exponential function with a user-input common ratio and did not feel that I had lost any generality.

QBasic is available for free and people are welcome to run the QBasic version of my simulation software on their computers. $u(s)$ is defined in a single function call and it can easily be modified beyond just assuming that $u(s)$ is exponential and asking the user to input the common ratio. QBasic is an easy language to learn. If a function call is insufficient, one can write a subroutine to define $u(s)$. Only the tail of $u(s)$ needs to be negative monotonic; that is, there exist an s^* such that, for all $s > s^*$, $u'(s) < 0$.

What happens when $u(s)$ is allowed to increase before diminishing? The demand distribution is defined as $c(m) = \sum_{r=0}^{\infty} c_0(x)$ with $x = \frac{u(0)}{u(r)} m$. Because $c_0(x)$ includes a logarithm, x must be positive, which it is even when $u(r) > u(0)$. But now the summation includes points on c_0 to the left as well as to the right of m. By Theorem 4, $\lim_{m \to 0^+} c_0(m) = 0$, so for small m we are including some negligible summands. Only if c_0 had a singularity at zero would weakening Axiom 2ii in this way be cause for concern, but it does not.

Modifying the axioms proposed by the founder of a science is how that science advances. For example, Leonhard Euler proposed three axioms to explain the ballistics of trench mortars:

1. Constant atmospheric density from the ground to the apogee.
2. Drag is proportional everywhere to the square of the speed.
3. Gravity is everywhere pointed downwards; e.g. the Earth is flat.

None of these three axioms are always true. But trench mortars do not go high enough for the air to become noticeably thinner. They do not fire at high enough speeds – 240 m/s is the limit – for drag to be proportional to higher powers of speed than two. And they do not fire far enough for it to matter that the Earth is a sphere.

The beauty of the axiomatic method is that the axioms can always be modified later to deal with more complicated situations. For instance, I developed an Android application for mortar fire control based on three similar but slightly modified axioms. The atmosphere becomes progressively thinner as altitude increases, as described by the axiomatic system proposed by Lewis Fry Richardson, which I take into account. And drag is proportional to the cube and then to the fifth power of speed at higher speeds, which I also take into account, as well as the effects of decelerating through the sound barrier (343 m/s).

Modern howitzers can fire on targets over the horizon and do take the curvature of the Earth into consideration, as well as many other things, like humidity, that have a negligible effect on mortar gunnery. But the theory employed by the modern artillerist is essentially that of Euler. If he could be resurrected and given the opportunity to talk to them, he would immediately recognize everything they are doing as being based on his 1745 annotated translation of Benjamin Robin's 1742 book, *New Principles of Gunnery.*

So it is no offense to me – indeed it is an honor – that Simon Gilson wishes to modify my axiom defining diminishing utility. I myself recently proposed a new axiom, that the parameter called importance, μ, has an exponential distribution, $\lambda e^{-\lambda\mu}$. Specifically, the inverse cubic law of large price fluctuations requires that the underlying distribution be $2e^{-2\mu}$. I prove this here.