PLANIMETRY OF ECONOMIC STATES

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ABSTRACT
The new information physical method of constructing the space of economic states is proposed. Unlike the existing theories of consumption, its properties are completely determined axiomatically by the operation of measurement and do not require phenomenological assumptions. We consider a transaction of exchange of valuables between two proprietors as such operation. The result of measurement is a dimensional number equal to the proportion of exchange. The constructed space appears to be Euclidean vector space with ordinary operators of composition of vectors, their scalar product, etc. The task of determining the parameters of equilibrium of a complex economic system can be formulated as a task of statics in the constructed space and can be solved by one of the physical methods.

Keywords: theory of consciousness, economic states, transaction, multidimensional space

INTRODUCTION

In the existing theories of consumer choice the main emphasis is made on modeling the consumer's preferences. In this relation, a number of problems connected with its idealization and the necessity of experimental definition of parameters of the model arise (Gilboa, Postlewaite & Schmeidler, 2010). At the same time, in physics the main properties of the space and the laws of motion in it are determined by the properties of symmetry of the procedure of measurement of distances and time intervals, rather then the properties of objects located in this space. Thus, for instance, the whole STR can be obtained only on the basis of the method of measurement of distances and time intervals using the light "meters" and "hours" formulated by A. Einstein. In the present paper we are following this particular constructive approach to the construction of space of economic states and studying of its properties. We are considering a transaction of exchange of property as a fundamental economic measurement. Natural properties of symmetry of this procedure allow introducing the operations of addition and scalar multiplication in a set of economic states. The obtained results allow predicting the relative value (and its variation in time) without introducing any additional assumptions on the consumer's preferences, but only on the basis of measurement of the projection of the vector of its state. Thus, the proposed approach allows excluding from the consideration the "hidden parameters" of the model of consumer, and rely only on the results of measurements of its state (concluded or rejected transactions). In the present paper we have limited ourselves to the discussion of application of the proposed model in the tasks of economic statics. We are first going to study the properties of the space of economic states generated by the method of their measurement. Then, on the basis of these properties, we are going to determine the notion of economic balance and formulate the laws of statics, and, finally, we are going to analyze the special aspects of motion of an economic analog of solid body in the space of states.

1. MULTIDIMENSIONALITY OF THE SPACE OF ECONOMIC STATES

We have previously proposed the physical methodology of constructing unidimensional models of economic systems (Tuluzov & Melnyk, 2010). The unidimensional space of states occurs as a consequence of the idealized assumption on the fact that in the process of exchange of a valuables A for B, B for C and C for A the following ratio is valid:

$$\left\{\frac{A}{B}\right\} \cdot \left\{\frac{B}{C}\right\} \cdot \left\{\frac{C}{A}\right\} = 1 \qquad (1)$$

where the symbol $\{A/B\}$ determines the result of measurement – the quantitative proportion of exchange of valuables A for valuables B. This assumption can be conditionally interpreted as a statement – "everything is measured only in money". Actually, if a fixed value in conventional units (money) is set for each valuable item, then it determines the proportions of its exchange for any other valuables. In this case, the ratio (1) is obvious. At the same time, in real economy it can be violated. Moreover, violation of this ratio actually represents the stimulus of every transaction! Each participant of the transaction assumes that his actions will be eventually profitable. At the same time, it becomes possible only in case of transition to the multidimensional space of states, in which each of the proprietors relies upon his own scale of values in the process of decision-making. In this case, in order that the transaction takes place, the two inequalities must be satisfied:

$$\left\{\frac{A}{B}\right\} > \left\{\frac{A}{C_A}\right\} / \left\{\frac{B}{C_A}\right\} \ ; \ \left\{\frac{B}{A}\right\} > \left\{\frac{B}{C_B}\right\} / \left\{\frac{A}{C_B}\right\} \qquad (2)$$

where C_A and C_B are the units of the scale of values of the proprietors A and B, respectively. It follows from these inequalities that the values A and B cannot be considered as scalars and that the proportions of exchange of two valuable items in a certain transaction can depend on the preferences of the participants of the transaction. Let us note that the main task of the present paper is not so much a formal generalization of the unidimensional space of economic states into a multidimensional state, but rather finding economic analogs of simple mechanical models and notions. The developed instruments of modeling will further allow analyzing the dynamics of complex real economic systems.

2. THE SPACE OF GENERALIZED ECONOMIC MEASUREMENTS

Let us call an ***elementary economic object*** (EEO) a certain valuable item, which looses or changes its properties in the process of its division into parts, and which belongs to a single ***proprietor***, i.e. a subject (a single person or a group of persons) who adopts decisions on its exchange for other valuables. Actually, we call the ***economic properties*** such properties, which determine the value of a specific object in case of its exchange. Let us assume that any other possible changes not effecting the results of transactions with this object do not change its economic state. According to N. Bohr, any physical measurement is based on the fundamental measurement – comparison with etalon. As a result of such measurement we can obtain only one of two possible results. Similarly, we consider the ***fundamental measurement in economics*** as an offer of transaction, for which a proprietor gives either a consent or a refusal. At the same time, each of the valuables can have an unrestrictedly large number of economically measurable properties. These properties depend on how it is intended to be used. Let us assume that object A is more valuable for proprietor X compared to object B. In this case the proprietor will refuse to exchange A for B. However, he can agree, for instance, to exchange A for 2B. In this case we can write down that $B_x < A_x < 2B_x$. In the classical economic model such measurements, similarly to physics, can be considered only as intentions, which do not effect the states of proprietors. Therefore, we can always perform an arbitrarily large number of similar measurements and find out the ratio of values of objects A and B arbitrarily accurately. If, for instance, $nB_x < mA_x < (n+1)B_x$, then we will consider that the attribute X of the object A is measured with the accuracy B_x/m. Such generalized economic measurements are usually the subjects of transactions in real economy. As a result of a ***generalized economic measurement*** we can obtain a rational number characterizing the proportions of a "fair" exchange of valuables A and B in relation to the consumer property X.

We can also consider A and B as different states of one ***universal economic object***, and associate ordered couple AB with an operator changing the properties for A to B. At the same time, it is unimportant if such changing of properties of an object is physically possible. Let us illustrate that the multitude of such changes forms a vector space. Let us associate the ***vector*** $\overrightarrow{AB}$ with any two states A and B, and the ***vector projection*** for the direction X with the proportion of exchange $\frac{A_x}{B_x} \approx \frac{n}{m}$.

Let us introduce the operation of ***composition of vectors*** as a consecutive execution of transformations of economic properties of the object corresponding to each of the composed vectors. If $\frac{A_x}{B_x} \approx \frac{n}{m} = P_{AB}$ characterizes the ratio of values A and B in relation to the property X and $\frac{B_x}{C_x} \approx \frac{m}{k} = P_{BC}$ - ratio of values of B and C, then $\frac{A_x}{B_x} \cdot \frac{B_x}{C_x} \approx P_{AB} \cdot P_{BC} = P_{AC}$. It is convenient to define the operation of composition of vectors in such a way that the projection of their sum on any direction would be equal to the sum of their projections on this direction. Therefore, we will consider the projection of vector $\overrightarrow{AB}$ on the direction (consumer property) X the number $x_{AB} = ln\, P_{AB}$. Then it follows from $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ that $x_{AB} + x_{BC} = x_{AC}$ for any X.

The zero element of the set of vectors is the formal transformation of the economic state, which does not change its properties. The negative vector in relation to vector AB is the vector BA. It corresponds to the reverse proportions of exchange. It is easy to see that for the discussed addition operation the axioms of the Abelian group are valid.

The operation of multiplication of the vector by a natural number n corresponds to the repetition of the corresponding transformation vector n times. The vector $\frac{1}{m}\overrightarrow{AB}$ corresponds to the reverse operation – division by an integral number. Multiplication of the vector by a rational number $\frac{n}{m}$ is determined in an obvious way on the basis of these definitions. For the operation of multiplication by a scalar determined according to this method the following set of axioms is valid: $\lambda(x + y) = \lambda x + \lambda y$; $(\lambda + \mu)x = \lambda x + \mu x$; $(\lambda\mu)x = \lambda(\lambda x)$; $1 \cdot x = x$. Thus, it can be stated that the set of various transformations of consumer properties together with the introduced operations of addition and multiplication by a scalar form a vector space.

So far we have been discussing vectors characterizing a pair of economic states of a universal economic object. If one of such states is selected as a **reference point**, each vector can be associated with properties of a certain real or hypothetical elementary economic object calculated in relation to this state. Thus, we can interpret the constructed space not only as a space of transformations of economic properties of universal economic objects - vectors, but also as a set of such properties - **points**. It allows us to call the space of generalized economic measurements the space of states of universal economic objects as well. Each pair of points-states corresponds to a transformation vector transforming a universal economic object from one state to another. Dynamically continuous changing of properties of a universal economic object is described by a ***trajectory*** in the discussed space.

3. SCALAR PRODUCT IN THE SPACE OF GENERALIZED ECONOMIC MEASUREMENTS

It is obvious that some consumer properties are interconnected. Accordingly, the proportions of exchange of valuables with such properties are interconnected. Let us consider the properties of the universal economic object, the changing of which is described by vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Changing by e times of the exchange proportion for the first of them corresponds to the shifting of the point of universal economic object by a unit of length along the distance $\overrightarrow{AB}$. At the same time, the projection of state of the universal economic object can also change on the direction $\overrightarrow{AC}$. If the consumer points corresponding to the selected directions are independent, then the projection on the direction of vector $\overrightarrow{AC}$ remains unchanged, and the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ will be considered ***perpendicular***. The number of mutually independent properties defines the spatial dimension of states of the universal economic object, and the corresponding vectors define its ***orthogonal basis***. Along with the reference point the basis forms the ***reference system*** in the space of states of the universal economic object. The result of measurements represents a number characterizing the proportions of exchange of the universal economic object in two states A and B. If this number is considered the number of projections on the direction in the space corresponding to the measurement, then its maximum value corresponds to the length of vector. We will accept it as the ***norm of vector***.

Formally, the norm in the vector space Rm is any representation $||\cdot||$, satisfying the following requirements at all values of x, y $\in R_m$ and $\alpha \in R$: 1)$||x + y|| \le ||x|| + ||y||$; 2) $||\alpha x|| = |\alpha|\cdot||x||$; 3) $||x|| = 0 \Leftrightarrow x = 0$. The second and the third requirement follow directly from the definition. The first (triangle inequality) follows from the requirement of possibility of arbitration in the discussed exchange procedures. Otherwise, it would be possible to receive riskless profit by cyclically making a sequence of exchanges $(A \to B \to C \to A \to \cdots)$. Thus, the constructed **space of economic states is metric.**

The definition of the projection of the vector $\overrightarrow{AB}$ on a certain direction set by a different $\overrightarrow{AC}$ and the norm of the vector $\overrightarrow{AC}$ is sufficient for defining the ***scalar product*** of vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, as the composition of these two numbers. The scalar product in R_m is the representation $(\cdot,\cdot)$ from $E \times E$ into R, satisfying the following axioms at all values of x, y, z $\in R_m$ and $\alpha \in R$: (1) $(x, y) = (y, x)$; (2) $(x + y, z) = (x, z) + (y, z)$;(3) $(\alpha x, y) = \alpha(x, y)$;(4) $(x, x) \ge 0$; (5) if $(x, x) = 0$, then $x = 0$. The properties (2-5) are satisfied on the basis of the definition. However, the fulfillment of axiom (1) – commutativity of the scalar product – is not obvious. We hereby postulate that it must be fulfilled due to the symmetry in case of absence of any additional influences on the consumers.

4. APPLICATION OF THE PYTHAGOREAN THEOREM AND TRIGONOMETRY IN THE SPACE OF GENERALIZED ECONOMIC MEASUREMENTS

The Pythagorean theorem in itself is a particular case of the general problem of calculating the length of the triangle side using its three known parameters. It only represents its most simple case, when the lengths of two orthogonal sides of the triangle (cathetus) are known. In economic models two pairs of generalized economic measurements of states AB and AC correspond to two such sides. In case of orthogonality of these directions for consumers that are ready to exchange A for B in the maximum proportion, A and C will be equal (and vice versa).

The length of each of the sections AB and AC is determined as the norm of the corresponding vector - the logarithm of the maximum possible exchange proportion of A for B or A for C, respectively. Following the Pythagorean theorem and measuring these lengths, we can calculate the maximum exchange proportion of B for C. The practical meaning of this result is not so essential, but the point of importance is that it allows an experimental verification.

In the general case, for calculating the length of the third side of the triangle the following formula is used:

$$\|\overrightarrow{BC}\| = \left(\|\overrightarrow{AB}\|^2 + \|\overrightarrow{AC}\|^2 - 2\left(\overrightarrow{AB} \cdot \overrightarrow{AC}\right)\right)^{\frac{1}{2}} \qquad (3),$$

where $\left(\overrightarrow{AB} \cdot \overrightarrow{AC}\right)$ is the scalar product of the vectors. In order to calculate it, we need to determine, for instance, in what proportions the consumers, which exchange A for C in the maximum proportion, are ready to exchange A for B. Further, on the basis of the measured scalar product of the vectors and their lengths we can calculate the cosine of the angle between the vectors and use the whole apparatus of trigonometry for solving more complex problems. The end result of their solution will be the prediction of projections - “fair” proportions of exchange for specific consumers. A more detailed analysis of the procedure of fundamental economic measurement with account of its influence on the proprietor’s consciousness allows us making an assumption that the space of economic states is finite-dimensional, that allows us limiting ourselves to a finite number of measurements for its description (Tuluzov & Melnyk, 2014). Thus, instead of estimating the utility value of a specific valuable item in conventional units on the basis of the phenomenological model of preferences, as it is normally done in the majority of existing models, we are considering only the measurable values.

In this relation, a natural question arises – what motivation can “force” the consumers to agree for the proportion of exchange of two valuables calculated in accordance with the trigonometric formulas. We assume that in the final analysis it is the law of absence of arbitration (possibility of receiving riskless profit) in classical economic systems.

5. ECONOPHYSICAL INTERPRETATION OF THE LAGRANGE MULTIPLIERS IN THE PROBLEMS OF ECONOMIC STATICS

On the basis of the aforesaid, we can consider the dynamics of any economic system as a motion of a set of elementary economic objects (analogs of material points) in the space of economic states. As it is illustrated in classical mechanics, the properties of such system are completely determined by the assignment of masses, kinematic links between the points and the potential function of their interaction. We have previously (Tuluzov & Melnyk, 2010) analyzed in detail the essence of the economic analog of mass. In this paper we are going to analyze in greater detail the economic analog of potential energy.

The task of statics is in finding the forces of interaction between the bodies in equilibrium position and equilibrium positions of such bodies. One of the most fundamental methods of solving such tasks is the principle of virtual displacements. It assumes that a certain function (potential energy), which in the equilibrium position possesses the extreme value, and limitations for possible displacements of these bodies (kinematic connections) are set. In this case the method of Lagrange allows calculating all the forces of reaction originating in the links, as well as the equilibrium values of the coordinates.

In economic applications a random objective function can be used as a Lagrange function. The limitations are also considered to be set a priori. It is considered that the form of the objective function is determined by the subjective choice of the proprietor and the limitations are determined by the external factors (Intriligator, 2002). At the same time, the mechanism of forming of the general objective function for several proprietors remains unclear. Besides, it is not always possible to determine the function of interaction between the proprietors connected with the conflict of interests. In this chapter we do not propose any new mathematical algorithms of solving the variational problem, but assume that the econophysical interpretation of the problem makes these issues considerably less critical. In the general case the objective function can depend on both coordinates and velocities, characterizing the state of economic system, and on time. Let us denote the objective functions for two economic systems as $V_1(\vec{r_1}, \dot{\vec{r_1}}, t)$ and $V_2(\vec{r_2}, \dot{\vec{r_2}}, t)$. So far as they do not interact, the equilibrium state of each of them is calculated independently. But when a certain connection is established between the systems due to technological dependencies or contracts concluded between proprietors, they should be considered as an integrated system of objects. Let us assume that the connection is a kinematic condition of the form $f(\vec{r_1} - \vec{r_2}) = 0$. In case of rigid connection (one of the companies is an exclusive supplier of raw materials for the other) this function degenerates into a δ-function. In mechanical systems such links are modeled using the ideal non-stretchable lines or absolutely rigid pivots. In the more general case in proximity to the equilibrium position this function can often be interpreted as a flexible connection with a certain coefficient of rigidity. In accordance with the method of Lagrange, in order to find the equilibrium position of the integrated system it is necessary to solve the variational problem: $\delta[V_1 + \alpha V_2] + \lambda \delta f = 0$. Besides the Lagrange multiplier λ, an additional coefficient α is introduced. It is required for coordination of dimensions of the economic analogs of potential energies V_1 and V_2, initially not interconnected. Similarly to physics, this coefficient can be determined experimentally in the analysis of results of interaction of the discussed systems. The additional summand λf has a physical meaning of potential energy conditioned by the interaction of economic objects. In this case, each of them is effected by a force from the other object, which can be calculated according to the following formula: $F_i = -\lambda \frac{\partial f}{\partial \vec{r_i}}$. Let us note that in case of the discussed form of kinematic condition (dependence only on the distance between the points) the analog of the third Newtonian law is automatically satisfied. Eventually, the application of the principle of virtual displacements in relation to economic systems allows calculating their relative equilibrium position and forces acting between them at the given objective functions $V_i(\vec{r_i}, \dot{\vec{r_i}}, t)$ and kinematic connections $f_{ik}(\vec{r_i} - \vec{r_k})$. By determining the Lagrange multipliers and forces, we can also define the notion of work as a scalar product of force and displacement.

The second summand αV_2 in the summary potential energy can be formally considered as a kinematic condition $V_2(\vec{r_2}, \dot{\vec{r_2}}, t) = const$ with a Lagrange multiplier α. Then the equilibrium state can be found in a space of a larger dimension on a hypersurface determined by the kinematic conditions $f(\vec{r_1} - \vec{r_2}) = 0$ and $V_2(\vec{r_2}, \dot{\vec{r_2}}, t) = const$. In the general theory of relativity a similar method allows considering the gravitational field as a curvature of the physical space – time. In the context of economic problems such method means that in certain cases a hidden kinematic connection of property with other economic objects can be considered as the reason of origin of an objective function of a proprietor. And, conversely, any new objective function can be realized by forming new kinematic links (rigid or flexible) between the proprietors. Thus, in the framework of the Lagrange method applied to the system of material points in the space of economic states, the distinctions between the subjective and objective components of the elementary economic object are eliminated.

By determining the Lagrange multipliers and forces, we can also define the economic analog of the notion of work as a scalar product of force and displacement.

6. DISPLACEMENT OF THE SYSTEM ECONOMIC SOLID BODIES IN THE SPACE OF STATES

In the process of modeling, the whole variety of mechanical systems, in some or other way, is limited to a number of idealized notions, which have analogs in economics as well. Thus, for instance, an absolutely solid body corresponds to a system of economic objects with unchanged distance between them. Such objects can be, for instance, companies interconnected by rigid technological links, or by an agreement on fixed division of total incomes.

Displacement of such economic solid body in the space of states is limited to its translational motion and rotation. The equilibrium position is determined by the requirements of the zero sum of external forces and the sum of moments of these forces. In this case any internal forces or links can be neglected. However, if the external forces and moments of forces effecting such system are absent or compensated, the system will displace with constant linear and angular velocity. The second effect occurs as a consequence of multidimensionality of the space of states and is perceived by external observes as harmonic oscillations of the system's cost relative to different axes (sets of consumer properties).

As it has been illustrated above, the forces $F_i = -\lambda \frac{\partial f}{\partial \vec{r_i}}$ correspond to economic links of the form $f(\vec{r_1} - \vec{r_2}) = 0$ and the third Newtonian law is observed. Besides, such forces between two bodies are directed along one line in the space of states. Therefore, the short-term interactions between the systems (collisions) can be described by the laws of conservation of momentum and the laws of angular momentum conservation, where the impulse is determined as the impulse of force ($dp = Fdt$) and the momentum is determined using the standard method.

Let us note that the economic analogs of the conservation laws do not require any additional derivation and are valid due to the condition of closedness of the system. Unlike the known attempts (Kitov, 2009) of phenomenological introduction of notions of force, impulse, work, etc. in economic models, we are considering their formal introduction on the basis of Lagrange method. This approach, in turn, will further allow studying the distribution of a large number of interacting economic objects and studying their behavior using the methods of statistical physics.

CONCLUSION

The offered method of constructing a multidimensional space of economic states is based on the previously developed algebra of fundamental measurements in economics and in the theory of consciousness. In the general case, the algebra of economic measurements can be limited to the theory of generalized selective measurement of Schwinger and requires the application of the mathematical apparatus of quantum mechanics. Nevertheless, even in the classical model the problems of statics (determination of equilibrium parameters of economic systems) can be solved using physical methods. For short-term interactions of economic systems the laws of conservation of momentum and the laws of angular momentum conservation are observed. Thus, we have obtained the rigorous substantiation of the possibility of considering the statics (and, in perspective, dynamics) of a random economic system as a physical system of interacting material points.

It allows further analyzing the behavior of their multitude using the methods of statistical physics on the basis of the strict fundamental approach, unlike phenomenological models. In the future, we are planning to generalize the constructed space of states for the quantum case (Tuluzov & Melnyk, 2012);(Melnyk & Tuluzov, 2014).

REFERENCES

Gilboa, I., Postlewaite, A. & Schmeidler, D. (2010). *The complexity of the consumer problem and mental accounting*. Mimeo: Tel-Aviv University.

Intriligator, M. D. (2002). *Mathematical optimization and economic theory*. Philadelphia : Society for Industrial and Applied Mathematics.

Kitov, I. (2009). *Mechanical Model of Personal Income Distribution*. WP/ECINEQ2009-110.

Tuluzov, I. & Melnyk, S. (2010). Physical Methodology for Economic Systems Modeling. *Electronic Journal of Theoretical Physics*, 7(24), 57-79.

Tuluzov, I. & Melnyk S. (2012). Algebra of fundamental measurements as a basis of dynamics of economic systems. arXiv:1209.1831v1 [physics.gen-ph].

Melnyk, S. & Tuluzov, I. (2014). Modeling in economics and in the theory of consciousness on the basis of generalized measurements. *NeuroQuantology*, 12(2), 297-312.

Tuluzov, I. & Melnyk, S. (2014). Manifestation of Quantum Mechanical Properties of a Proprietor's Consciousness in Slit Measurements of Economic Systems. *NeuroQuantology*, 12(3), 398-411.