Relativistic Theory of Value
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Abstract
On the basis of the theory of fundamental measurements and the relativistic hypothesis on the absence of a dedicated system of values, a space of 1+1 dimension has been constructed, in which each point is associated with a value (object of possible transaction). An illustrative “model of paints” and their mixtures, the values of which correspond to vectors in this space, has been proposed. It has been shown that the transitions from one system of values to another are described, similarly to physics, by the Lorentz transformations. In the proposed model, all classical relativistic effects are present. For inertial motion in the space of models, the principle of maximum benefit has been formulated, which represents an analog of the principle of least action. In the “model of paints”, the value analog of a homogeneous gravity field has been considered, and the simplest problem of dynamics in this field has been solved. The perspectives of generalization of the developed model for the space of 3+1 dimension and the quantum analog of such space have been analyzed.

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**References**

**Introduction**

The notion of value can be applicable to various objects, phenomena and processes of the surrounding world. Thus, we can speak of value or valuelessness of life, value of obtained and lost information, value of relations between people, their rights and obligations, things they possess, etc. However, such evaluation usually does not extend beyond subjective emotional characteristics. The only domain of human knowledge, in which scientists attempt to give a quantitative evaluation of the notion of value, is economics. However, even here this evaluation is limited to determination of market price of a certain object. The majority of economic models of price formation are phenomenological. They are based on a certain “obvious” set of
properties of the seller and buyer, as well as on the classical mathematical apparatus of description of the obtained system.

Following this approach, we have developed [1] the phenomenological model of a company, and have shown that in case of observance of natural economic requirements of “rationality”, its trajectory in a classical one-dimensional space of value can be described by the laws of classical mechanics. We have introduced the economic analogs of dynamic variables in physics and analyzed particular cases of motion of closed systems in a classical space of values. This study has been awarded by the Majorana Prize and in many aspects determined the direction of our further researches. However, it appeared that the correct introduction of the laws of dynamics of this “motion” requires not only the construction of the value coordinate space, but also the introduction of a value analog of the time scale on the basis of the analysis of transaction properties considered as fundamental measurements [2].

Previously, we have shown that in case of a more rigorous information-measurement approach to the formation of cost of goods, we can refuse from notions on the existence of a dedicated scale and obtain a completely relativistic space of prices in a certain economic system [3].

In the present study we are going to generalize the previously introduced notions and to demonstrate that the relativistic space of value originates in all systems, in which a set of fundamental measurements can be introduced and their maximum symmetry can be required.

In this interpretation the economic, physical and information spaces can be considered as particular cases of the general approach to the description of the results of fundamental measurements [4, 16]. This allows easily transferring (adapting) the results obtained in the process of studying of special phenomena in the physical space for the economic processes.

Actually, this task is considered the basic problem in modern “econophysics”. To a considerable degree, many of the existing econophysical analogies have already been studied and described in numerous papers [5]. However, they are of phenomenological character, which significantly limits the possibilities of their implementation.
In the present study we have made an attempt to eliminate this obstacle and analyze the fundamental mathematical principles, which can allow transferring the physical laws into the world of economic objects and phenomena. Rigorous construction and studying of the properties of the relativistic space of values is the first step in this direction.

**I.1. Value as a measurement category**

The main objection quoted by the supporters of phenomenology in the “theories of value” is the fact that not everything can be considered an object of sale in this world. We are not going to argue with this opinion. Considering a sale as an indirect (money) exchange of one value for another, we must of course admit that there are “priceless” things for a human, which he never agrees to sell or to exchange for anything else.

However, in case of a rigorously mathematical approach it turns out that even these situations (unconditional refusal from exchange) can also be described by the categories of value. Moreover, any situation of choice of a human can be formalized as a solution of a variational problem in accordance with a certain criterion of optimality. This criterion of choice represents the subjective notion of values. Strictly speaking, both Giordano Bruno and Galileo made their choice in accordance with their own subjective scale of value. In this paper we are not going to “judge” their choice, we are rather going to try to understand what mathematics can describe the situation of choice.

We are to admit that practically all philosophic, religious and worldview concepts offer a certain scale of values, in accordance with which the choice is possible. At the same time, they do not even try to harmonize their outlooks. Actually, the argument on what is more valuable and “worth living for” is limited to a banal dispute on which axiomatics is better.

However, any axiomatics only defines the language, on which the observed events will be described. The question of which language is easier for description is arguable. But putting the question of which language is better is absurd. It is just necessary to define the rules of “translation” from one language into another instead.
This is what physics came to during the many years of its development. This is the approach we are going to develop in application to the theory of value. At the same time, we are not going to distinguish any specific criterion of evaluation as the “most correct”. Instead, we are going to find invariants which retain their value at any given criteria.

B 2. Fundamental measurements – transactions of exchange of values

We have previously introduced the notion of fundamental economic measurements, which we proposed to consider as the results of elementary (unconditional) transactions on exchange of values [6]. The results of such measurement can have only two values – consent for a transaction or refusal of a transaction. This situation corresponds to the statement of Niels Bohr on the fact that any measurement is based on the comparison with an etalon. Further, we have introduced the operations of addition and multiplication for such measurements. The first of them is equal to the logical operation “or” for the information obtained as a result of measurements, and the second one is equal to the operation of non-commutative “and”.

In particular, we have previously shown that the calculation of the results of a fundamental economic measurement (transaction), represented as an algebraic function of other (elementary) transaction requires the use of the quantum-mechanical formalism instead of the classical formalism. The axiomatics and algebra of the fundamental economic measurements is in many aspects analogous to the generalized measurements introduced by Schwinger [7], however, it does not require preliminary assumptions about the measurements, and is based only on the properties of their symmetry.

In this study in the process of discussion of the fundamental measurements we will focus on the mathematical structure of the relations generated by such measurements. At the same time, we will analyze only their classical approximation, considering that the result of any fundamental measurement (transaction) is unambiguously determined by the position and velocities of the exchanged objects in the economic space of states.
**B.3. Relativistic principle in the theory of value**

One of the fundamental experiments in the history of physics is considered the Michelson-Morley experiment. Despite the fact that there are still several alternative interpretations of its results, actually, it has been proven that no method of locally (without leaving the premises of the laboratory) determining the velocity of its motion relative to the rest of the universe exists.

Generalizing this conclusion for any similar measurements, we can assume that by performing measurements (performing transactions on exchange or refusing of such transactions) in a local system of values, it is impossible to determine their value relative to the rest of the objects or essences.

Hereof it follows that any scale of values can be accepted at the basic, and the laws of transformation of relative values (results of transactions) from one scale to another must not depend on this selection. We are not going to “argue about personal preferences”, but we are going to compare them.

In the physical theory the existence of a dedicated “absolutely stationary” reference system is associated with the hypothesis of the “world ether”, while the notion of the “center of the universe” is associated with the hypothesis of its boundedness. Rejection of these hypotheses is mathematically reflected in the fact that both in classical and relativistic physics the laws of motion are derived only for relative coordinates and velocities. Similarly, in the theory of values we will require that no “absolute value” will exist. It will allow to a significant extent determine the laws of transformation of values from one reference system into another, only from the considerations of symmetry.

**B.4. Criterion of comparison of values – choice**

In the economic theory we are usually speaking of measurable values of commodities. However, in the general case, we can face a situation when a subject is forced to select one of two or several alternatives of behavior. In the fundamental economic measurements it represents the choice between the consent for a transaction or refusal of it. For this purpose the subject compares the price of the exchanged commodities.
We will generalize the notion of price for the notion of value. Let us consider that in the rest of the situations of choice the subject also compares the value of its state in each of the alternatives and selects one of them on the basis of a certain subjective criterion of value.

At the same time, we do not set up the task of examine the mechanisms of this choice. Similarly to physics, there is absolutely no need to understand the operation of a clock if it works “correctly”.

Instead, we will focus on determination of those simple requirements to the value “clock”, which actually define the properties of the value “time”, i.e. we will determine the criterion of choice (and the subjective value of a specific object) on the basis of the subject’s behavior instead of his senses or thoughts.

The concept of such attitude to the description of the surrounding reality has been previously formalized by us at the “information-measurement approach” [8]. Rene Descartes, who has formulated its basic provisions, is rightfully considered its originator [9]. This approach contributed to the avalanche-like development of physics, and, later, other exact sciences in the second half of the 17th century. We hope that a similar “revolution” is possible in relation to the rest of (non-physical) essences; in particular - to the notion of value.

1. **Model of paints (MP) – the simplest illustration of a set of objects with comparable values**

1.1. **Model description**

Before introducing economic or information dimensions and constructing the space of states of the observed objects, let us consider the simplified (illustrative) model of value of a certain set of objects. Let us note that in the process of development of this model we pursued the aim of its maximum simplification, leaving the minimum required number of variable parameters. Let us consider a set of mixtures obtained from paints of two pure colours: red “R” and yellow “Y”.

The colour of mixture is determined by their proportion and the quantity – by the volume measured in certain conventional units (for instance, in liters). From mathematical point of view, this set of objects represents a convex one-dimensional
set, which can be associated with a straight line segment: \( v \in [-1; 1] \). In this case the ratio of quantities of «R» and «Y» paints in the mixture will be the following

\[
\frac{V_Y}{V_R} = \left( \frac{1+v}{1-v} \right).
\]  

(1)

Both the colour and the volume of any mixture of «R» and «Y» paints are unambiguously determined by their quantity. Therefore, it is natural to represent the set of such mixtures as a set of points on a plane with coordinate axes, in which the volumes of paints forming the mixture are indicated (Fig. 1).

To each mixture a vector can be assigned. In case of mixing of two mixtures the quantities of the red and yellow paint are added independently of each other. Therefore, such mixing corresponds to a sum of vectors. For instance, in case of adding of the pure paint «R» to the mixture «A», it becomes mixture «B» with a different quantity of paint and a different colour (Fig.1).

Fig 1. Set of mixtures of two pure paints «R» and «Y» fills the quadrant of plane. Rotation of the axes of coordinates allows proceeding to their quantitative-colour representation.
The lines $V_R + V_Y = V$ correspond to the total volume of mixture and the value 

$$\frac{V_Y - V_R}{V_R + V_Y} = v$$  \hfill (2)

corresponds to its colour. Therefore, rotation of the coordinate axes at $45^0$ (Fig.1) allows proceeding to their quantitative-colour representation. Vector projection on the axis $V$ determines the total quantity of paints in the mixture (in liters), and projection on the axis $x$ determines the difference (in liters) between the quantity of «R» and «Y» paints. Then the colour $v$ can be defined as the tangent of angle between the vector of mixture and the axis of quantity $V$. At that

$$V_A = V_{Y(A)} + V_{R(A)}; \quad x_A = v \cdot V_A = V_{Y(A)} - V_{R(A)}$$  \hfill (3)

1.2. Objective criterion of comparison of values in the model of paints

So far, we discussed only the physical quantity of “R” and “Y” paints in the mixture. In order to proceed to the evaluation of their value it is necessary to introduce the criterion of comparison of values of two mixtures. Obviously, it may depend on many subjective factors, consideration of which is beyond the scope of the constructed model. At the same time, it is possible to determine the objective criterion of comparison of values. For this purpose we will consider that

- Until the moment of painting the components of the mixture – paints “R” and “Y” – are stored separately and can be divided into any parts;
- Any part of the mixture or pure paint has a positive value.

These simple and natural assumptions are sufficient for introducing the absolute criterion of value:

(*) We will consider the mixture «A» absolutely more valuable compared to the mixture «B», if the quantity of each of the pure paints in it is not less than in mixture «B».

In other words, if the mixture “A” contains the mixture “B” as its component, then it is absolutely more valuable.
The point of principal importance is that this criterion is not associated either with the units of measurement of value of pure paints, or with the fact to what extent one of them seems more valuable compared to the other. The ratio of absolute comparison of values introduced by us represents the relation of inclusion at a set of mixtures of paints. For each specific mixture “A” (element of this set) the rest of the elements can be divided into 3 subsets:

- Mixtures, which are contained in “A” and therefore are absolutely less valuable compared to it;
- Mixtures, which contain the mixture “A” and therefore are absolutely more valuable compared to it;
- The rest of the mixtures, the value of which depend on the fact to what extent the paint “R” is more or less valuable compared to the paint “Y”.

Thus, the set of all mixtures can be divided into 3 parts (Fig. 2). Anticipating the further analogy with the physical space, let us name the corresponding domains in the vector space of mixtures:
- the lower «cone» of the mixture «A» (mixtures of paints, the value of which is absolutely smaller compared to the value of “A”),
- the upper «cone» (mixtures of paints, the value of which is absolutely larger compared to the value of “A”),
- the domain of value-like mixtures – those, which can turn out to be both less valuable and more valuable compared to the mixture “A”.

1.3. Scales for evaluation of value in the model of paints

From the above mentioned definition, it follows that if the mixture “B” is absolutely more valuable compared the mixture “A”, then it completely contains “A” and a certain non-zero mixture, which we will define as $(\Delta_{AB})$. Then we can construct such a sequence of mixtures $A_i$, that $A_{i+1} = A_i + \Delta_{AB}$ (Fig.2). Let us note that the constructed scale can be made continuous, by supplementing it with interim counts and by introducing the operation of division of the mixture into a whole number of fractions. Thus, we can state that any pair of values connected by the absolute relation can generate a scale of measurement of value of the rest of the objects.
For using this scale it is additionally necessary to define the operation of projection on this scale of the value of any object $B_j$, which is not located on the axis $A_i$. In other words it is necessary to define the rule, which unambiguously determines the quantity of the mixture $A_i$ in the scale «$A$», equal to the mixture $B_j$ (Fig.2).

Fig 2. Diagram for determining the absolute and relative ratios of value, as well as the scale of values.

2. Relativity principle in the model of paints

The ratio of values of mixtures of different colours is either absolute, or depends on the ratio of values of pure paints “R” and “Y”. Each of the possible scales of value in the model of paints is characterized by a certain colour of mixture $\langle \Delta_{AB} \rangle$. If we combine the zero reference point of the scale with the 0-th quantity of paints, then all points on the scale will correspond to one and the same colour, the same as the colour $\langle \Delta_{AB} \rangle$. The ratio of volumes of pure paints “R” and “Y” in the mixture $\langle \Delta_{AB} \rangle$ allows associating this scale (and this colour) with a proper relation of value of the paints “R” and “Y”.

(***) We will assume that the value of pure paints, contained in a mixture of a certain colour, is identical for the scale of values constructed on the basis of this mixture.
For instance, if the quantity of red paint in the mixture $\Delta_{AB}$ is $n$ times larger than the yellow paint, then in case of projecting on this scale we consider that the red paint is $n$ times less valuable. Although the above mentioned axiom does not require proving, let us present two arguments “in its defense”.

2.1. Physical analogy of the relativity principle in the model of paints

The scale of value in the model of paints corresponds to the scale of time in the inertial reference system in physics. Colour, written down using the parameter $v$, corresponds to velocity (in C units) of this reference system, while the quantity of pure paints “R” and “Y” contained in this colour corresponds to the time of propagation of a light beam in both directions in the process of measurement of the time interval. Thus, the requirement of equivalency of the quantity of pure paints is equal to the requirement of equal velocity of light in the process of its propagation “there and back”.

Besides, the equivalency of the quantity of pure paints in a mixture means the value neutrality of its colour. It is an analog of a stationary state of a reference system in physics. Thus, the accepted axiom is equivalent to the physical statement on the absence of a dedicated system of absolute rest.

2.2. Scale of value in the model of paints as an analog of the Einstein clock

Let us note that the quantitative comparison of volumes of paints in a mixture is possible as long as we continue evaluating it using not only value, but also physical characteristics (volume in liters, for instance). But as soon as we reject the results of physical measurements, only the possibility of relative comparison of values will be left.

It means that in none of the reference systems (scale introduced above) in a generalized space of values of mixtures we will be able to evaluate who many times the paint “R” is less or more valuable compared to the paint “Y”. Instead, we can only compare the relative values of “R” and “Y” in two different reference systems, only if we know how these systems move relative to each other.
At the first glance, such a strictly value-based consideration of mixtures complicates the process of their description, compared to the initial (physical) method. However, in case of generalization of the information-measurement approach for arbitrary economic systems we are forced to make this step.

Let us demonstrate that a similar problem occurs in the physical relativistic space-time as well. All space-time measurements in the relativistic mechanics can be reduced to the measurements of the moment of transmission of a light signal and the moment of reception of this signal, reflected from the observed object. For measurement of these moments, a light clock is additionally created, consisting of two parallel mirrors, between which the light beam propagates (Fig.3). We will call it the “Einstein clock” (EC). However, the result of such measurements is completely determined by the changing of distance between the mirrors. In physics, the Einstein clock rate represents a sequence of “tick-tacks”. At the same time, the interval of the “tick” (axis B in Fig.3) is equal to the time of propagation of the light beam from left to right between the mirrors, while the “tack” (axis A) – from right to left.

It is obvious that for any pair of uneven and not interconnected sequences of sections created on axes A and B, a corresponding Einstein clock exists, in which the distance between the mirrors and their trajectory change in a corresponding manner (two blue lines in Fig.3). Therefore, the information on time intervals measured according to “own” clock of the observer does not indicate the motion of the observed object until we ensure a constant distance between the mirrors of its Einstein clock.

However, it appears impossible, as the relativistic effect of shortening of distances in moving reference systems and the curvature of space in the field of gravitation are not connected with the properties of the matter. The only option in this situation is to coordinate the distances between the mirrors of the Einstein clocks of different observers or one observer in different moments of time. Actually, these procedures of synchronization, as well as the requirements of symmetry to their results, to a significant extent determine the laws of the special theory of relativity and the general theory of relativity. In this and further studies we are planning to travel step-by-step this entire path and eventually obtain the laws of motion in the economic space analogous to the physical laws.
In the discussed model of paints the role of the Einstein clock can be performed by two phases of the technology of painting. The first phase (“tick”) corresponds to the application of the thin layer of paint “R” (in physics – travel of the light signal from the first to the second mirror). The second stage corresponds to the application of the thin layer of paint “Y” (in physics – reverse travel of the light beam). Only after that the paints are mixed and dry out, creating a specific colour of the painted product. Similarly to physics, the obtained colour does not depend on the thickness of layers (distance between the mirrors), but only on the ratio of their thicknesses.

The economic essence of such representation has only a finite result – prediction of the results of a certain transaction on the basis of the results of other transactions with the same colours and mixtures, which are already known. The result of such prediction in the model of paints can be influenced not by the physical thicknesses of the layers (measured by a thickness gauge, for instance), but by their relative value in the process of exchange for a mixture with different thicknesses of layers. In other
words, in the economic space, for determining the colour of a mixture, we are to compare the value of the layers of paint rather than their thicknesses.

As the relative value of “R” and “Y” paints can be selected randomly (for each of the workshops it may be different), then the colour (analog of velocity) in this representation becomes a relative magnitude as well. In this case, we cannot tell how “red” this or that colour is. However, we can tell to what extent it is more red compared to a different colour, which we consider a “reference”. In physics, the analog of the “reference” colour is the inertial reference system, the velocity of which we take as zero.

2.3. Lorentz transformations as an instrument of transformation of scales of values

In the special theory of relativity the space-time coordinates of an event measured in two different inertial reference systems are interconnected only by the Lorentz transformations. We will demonstrate that these transformations allow associating two different subjective evaluations of value of a certain random mixture in the model of paints. At the same time, we are not going to use any physical analogies or additional assumptions, besides the aforesaid theses.

In case of taking into account the volume of paints, a dedicated scale (colour) is used, in which equal quantities of “R” and “Y” paints have the same value. In Fig.4 it is represented by the vertical axis $X_0$. All mixtures of the volume $\alpha_1$ are indicated by vectors, projection of which on axis $X_0$ corresponds to a corresponding point of the axis $X_0$. For instance, the mixture $\alpha_1 X_1$ in this reference system has the coordinates $(v_1 \alpha_1; \alpha_1)$, where $v_1$ is defined from (1). Our task is to determine the coordinates of this mixture relative to the axis of colour $X_2$, which is described by the parameter $v_2$.

For a random pair of prices of pure paints $S_R(\$/l)$ and $S_Y(\$/l)$ we can draw isolines of equal quantities of mixtures. They connect the points on axis “R” and “Y” corresponding to equal volumes of these paints («1 conventional unit» and isolines parallel to it for the price ratio $S_Y/S_R = 5/9$ in Fig.4). To each price ratio $S_Y/S_R$ corresponds a specific colour of mixture, at which its “R” and “Y” components have the same value. In Fig.4 it is indicated by the colour $X_2$. 
For any mixture (for instance, $a_1X_1$ in Fig.4) the quantity of equal value of mixture of colour $X_2$ can be determined using the corresponding isoline drawn through the end of vector $a_1X_1$. The point of its crossing with the scale $X_2$ appears to be the point of crossing of the diagonals in the rectangle with the vertexes $a_1X_1$, $a_2X_2$, $a_3X_2$. Therefore, the quantity of equal value of the mixture $X_2$ can be calculated as a semi-sum of the corresponding limit prices of exchange of this mixture on the scale $X_2$. In physics, a similar property (calculation of distance as a semi-difference of time moments of sending of a light signal and reception of a reflected signal) is interpreted as equal velocity of light in both directions in an intrinsic reference system.

The distance on the axis of quality can be calculated as the length of the semi-diagonal line connecting equivalent events, i.e. as the semi-difference of the limit prices of exchange. However, the obtained values of quantity of the mixture $\left(\frac{a_2 + a_3}{2}\right)$
and the distance to it on the axis of quality $\left(\frac{a_3-a_2}{2}\right)$ are expressed in the units of value of the scale $X_0$ (in liters). For proceeding to evaluation of these values according to scale $X_2$ it is additionally necessary to synchronize the units of measurement of value of these scales (tick-tacks).

2.4. Synchronization of scales of values

Let us note that the mixture $a_3X_2$, for instance, is equal in value to the mixture with the volume $a_3$ liters according to the scale $X_0$. However, relative to the values of the scale $X_2$ of this mixture a smaller quantity of $a_3^*$ liters of the mixture $X_0$ is equal in value. A similar paradoxical conclusion can be made in the process of analysis of simultaneous events in two inertial reference systems in physics. For its explanation the principle of relativity is used (absence of a dedicated reference system), requiring that the lag of the clock of a moving reference system compared to the clock of a stationary reference system would not depend on the fact, which of the systems we consider stationary. Acting according to the same procedure, we will require that the ratio $a_3/b_3 = b_3/a_3^* = \gamma$ would be valid. A similar method of synchronization of clocks in the special theory of relativity is called radio-location [10], and the parameter $\gamma$ is called the Lorentz factor.

From geometric ratios (Fig. 4) it follows that $a_3^*/a_3 = 1 - v \cdot \tan(\varphi - \pi/4)$, $\tan(\varphi) = \left(\frac{1+v}{1-v}\right)$. Then $\tan(\varphi - \pi/4) = \left(\frac{1+v}{1-v} - 1\right)/\left(\frac{1+v}{1-v} + 1\right) = v$ and $\gamma^2 = a_3^*/a_3 = 1 - v^2$. Thus, from the symmetry of the scales $X_0$ and $X_2$ it follows that the “value-based clock’s rate is slower” in a moving reference system in

$$\gamma = \sqrt{1 - v^2} \quad \text{times.} \quad (4)$$

These ratios are sufficient for obtaining the Lorentz transformations and the law of addition of colours for a random mixture $a_1X_1$ for transition from one scale of values to another. Further, we will propose a more simple derivation of this formula.

It was previously shown (for instance, in [11]), that the postulate on constant velocity of light is not necessary for deriving the Lorentz transformations and the basic effects of the special theory of relativity. It follows from our conclusion that these transformations can be obtained without consideration of any motion, but only
on the basis of the principle of relativity (absence of a dedicated colour of mixture in the model of paints).

3. Relativistic effects in the space (1+1) of the model of paints

The common practice in physics is to associate the relativistic effects with the postulate on the constant velocity of light. Considering the mirrors in the Einstein clock, moving in a specific manner, and requiring constant velocity of light in different reference systems, we will be able to illustrate most of the qualitative effects of the theory of relativity. Let us consider the manifestations of these effects in the model of paints.

3.1. Law of «addition of colours»

We have previously defined the parameter of colour as the analog of physical velocity (in units of measurement of the velocity of light) (2). Suppose a certain colour is characterized by the parameter \( v_1 = \frac{V_{1Y} - V_{1R}}{V_{1R} + V_{1Y}} \), and the other - \( v_2 = \frac{V_{2Y} - V_{2R}}{V_{2R} + V_{2Y}} \).

Let us note that in both cases we evaluate the mixtures in the dedicated (physical) reference system, in which we take account of their volumes instead of values. Actually, it means that in this reference system the equal volumes of “R” and “Y” paints are considered as having equal value. In order to calculate the colour of the second mixture relative to the first one, first it is necessary to determine the relative value \( S_{R21} \) and \( S_{Y21} \) of pure paints contained in it relative to the volumes of these paints in the first colour.

At the same time, in physics the first reference system is considered stationary. A in the model of paints is considered equal to a certain \( S_0 \) of the value of volumes of “R” and “Y” paints contained in the first mixture. Then

\[
S_{R21} = \frac{V_{2R}}{V_{1R}} S_0 ; \quad S_{Y21} = \frac{V_{2Y}}{V_{1Y}} S_0.
\]

\[
v_{21} = \frac{S_{Y21} - S_{R21}}{S_{R21} + S_{Y21}} = \frac{V_{2Y}V_{1R} - V_{2R}V_{1Y}}{V_{2R}V_{1R} + V_{2Y}V_{1Y}}
\]

\[
\frac{v_2 - v_1}{1 - v_2 v_1} = \frac{V_{2Y} - V_{2R}}{V_{2R} + V_{2Y}} \frac{V_{1Y} - V_{1R}}{V_{1R} + V_{1Y}} = \frac{V_{2Y}V_{1R} - V_{2R}V_{1Y}}{V_{2R}V_{1R} + V_{2Y}V_{1Y}}
\]
From which we obtain the relativistic law of addition (in this case – subtraction) of colours
\[ v_{21} = \frac{v_2 - v_1}{1 - v_2 v_1}. \] (6)

Transition to the classical (not relativistic) limit of the formula (6): \((v_{21})_{RI} = (v_2)_{RI} - (v_1)_{RI}\) is possible at values of colours \(v_2\) and \(v_1\) close to zero. Thus, in the classical limit we can only speak of the relative value of shades of the basic colour. In this case in all three considered mixtures the ratio of quantity of “R” and “Y” paints is almost identical.

3.2. Effect of reduction of qualitative differences (analog of length) in a moving reference system

In physics the effect of reduction of length is observed in case of a solid rod moving in the direction of the velocity of motion relative to the observer. The trajectory of the tips of the rod in the intrinsic reference system represents two lines parallel to the axis of time. This effect may not be associated with the changing state of the rod, but can be explained by changing of the scale used to measure this length instead.

In the model of paints, as an analog of a solid rod, we can consider 2 sets of mixtures, in each of which the quantity of the “R” paint is different from the quantity of the “Y” paint for a constant value, independent of the total volume of mixture (red lines in Fig.5). Such pair of sets of mixtures of paints can play the role of a “clock”, analogously to the Einstein clock. Let us note that in this case the colour of both mixtures changes in case of increase of their volume, but the addition as a result of changing of the total volume always contains equal quantity of “R” and “Y” paints (the colour, which can be conditionally considered as 0-th).

In case of transition to the reference system of values, associated with the colour \(X_2\), the qualitative difference (analog of distance) between equivalent states of two mixtures is equal to the length of the diagonal of a rectangle (marked in yellow in Fig.5). It is equal to the length of the other diagonal, equal to the sections of the scale \(X_2\) (marked in black in Fig.5). Thus, the qualitative difference (analog of distance) between the mixtures measured according to the scale \(X_2\) is equal to the length of the
section $\Delta V_2$ according to this scale, while the difference measured according to the initial (physical) scale $V(l)$ is equal to the length of the section $\Delta V_0$.

![Diagram]

Fig.5. Reduction of qualitative differences between two mixtures in the process of transition to measurements relative to a mixture of a certain “non-zero” colour (analog of the moving reference system in physics)

As we have shown above, the “value-based clock” of the scale $X_2$ indicate in $\gamma = \sqrt{1 - v^2}$ a smaller interval of “time” between the pints A and B compared to the “clock” of the scale $V(l)$. Therefore, the qualitative difference between the two mixtures turns out to be $\gamma$ times smaller. It means that in case of transition to the evaluation of values of two mixtures relative to the scale $X_2$ (different colour of the mixture), their equal quantities will, as previously, differ by the constant difference of the price of volumes of “R” and “Y” paints forming these mixtures. At the same time, this difference will be $\gamma$ times smaller compared to the constant price difference between them in the initial reference system.

**3.3. Analog of interval between two mixtures as an invariant value**

For calculation of extremum proportions of exchange of mixtures of two different colours it is necessary to know the portion of paint “Y” in these mixtures
(values \(x_1\) and \(x_2\) in case of changing of physical volumes). However, in case if we exclude the possibility of measurement of its quantity in liters, these values become indeterminate. They depend on what “conditional units of value” we use to measure the quantity of “R” and “Y” paints in the mixtures. Let us denote the unknown volumes of these units in liters as \(\langle R_1 \rangle \text{ and } \langle Y_1 \rangle\). Then the portion of paint “R” in the mixture \(X_1\) measured in these units will be:

\[
x_1^* = \frac{n_R}{n_R + n_Y} = \frac{\frac{V_{x_1}}{R_1}}{\frac{V_{x_1}}{R_1} + \frac{V_{1-x_1}}{Y_1}} = x_1 \frac{Y_1}{Y_1 x_1 + R_1 (1-x_1)}.
\]

Correspondingly:

\[
x_2^* = x_2 \frac{Y_1}{Y_1 x_2 + R_1 (1-x_2)};
\]

\[
(1 - x_1)^* = (1 - x_1) \frac{R_1}{Y_1 x_1 + R_1 (1-x_1)}; (1 - x_2)^* = (1 - x_2) \frac{R_1}{Y_1 x_2 + R_1 (1-x_2)}
\]

\[
S_{12}^{* \text{max}} = S_{21}^{* \text{min}} = \frac{x_1^*}{x_2^*} = \frac{x_1}{x_2} \frac{Y_1 x_2 + R_1 (1-x_2)}{Y_1 x_1 + R_1 (1-x_1)};
\]

\[
S_{21}^{* \text{max}} = S_{12}^{* \text{min}} = \frac{(1 - x_2)^*}{(1 - x_1)^*} = \frac{(1 - x_2)}{(1 - x_1)} \frac{Y_1 x_1 + R_1 (1-x_1)}{Y_1 x_2 + R_1 (1-x_2)}
\]

Let us consider the product of the maximum and minimum exchange prices of a certain quantity of mixture \(X_1\), measured according to the same economic clock (in the reference system \(X_2\) in Fig.4). In physics these prices correspond to a light signal emitted in the reference system \(X_2\), reflected from the observed object and returned back.

\[
S_{12}^{* \text{max}} \cdot S_{12}^{* \text{min}} = \frac{x_1^*}{x_2^*} (1 - x_2)^* = \frac{x_1}{x_2} \frac{(1 - x_2)}{(1 - x_1)} = S_{12}^{\text{max}} \cdot S_{12}^{\text{min}}
\]

As a result we obtain a product of two relative prices invariant in relation to the selection of the units of measurement. In the intrinsic reference system \(X_1\) the maximum and the minimum prices coincide and their product is equal to the square of value of the mixture in this reference system.

Let us note that any changing of the state of the observed mixture \((X_1 \rightarrow X_1')\) results in a corresponding changing of the measured prices. It can be shown that the product of these changes also represents an invariant dimensionless value:

\[
(S_{12}^{\text{max}} - S_{12}^{\text{max}}) \cdot (S_{12}^{\text{min}} - S_{12}^{\text{min}}) = \Delta S_{12}^{\text{max}} \cdot \Delta S_{12}^{\text{min}} = \Delta S_{12}^{* \text{max}} \cdot \Delta S_{12}^{* \text{min}} = \text{inv} = (s_{12})^2
\]
In physics this value represents an interval between two events \((X_1 \rightarrow X_1')\). It can be written down in the following form:

\[(s_{12})^2 = (V_{12})^2 - (x_{12})^2\]  

(7')

Similarly to physics, in the model of paints \((s_{12})^2\) can possess both positive and negative values. In the first case it means that there will be a proprietor (subject), for which the discussed mixtures are indistinguishable in quality, but for all subjects one of them will be more valuable compared to the other (analog of the time-like interval). In the second case we can speak of the imaginary value \(s_{12}\). In this case there will always be a subject, for which the discussed mixtures will be of equal value, but for all subjects they will be different in quality (analog of the space-like interval).

### 3.4. Paradox of «twins» in the model of paints

In the well-known in physics “paradox of twins” it has been shown that for all possible trajectories of motion of clocks from one point to the other within a fixed interval of time, the maximum time reading will be indicated by the clock moving in the direct line with a constant velocity (in the particular case of coincidence of the initial and the finite points – for a stationary observer) (Fig.6).

In proving of this effect, we use only the kinematic effects which can be completely described by the Lorentz transformations. Additionally, we use the “obvious” statement that a clock moving with acceleration, locally has the same rate as a clock inertially moving nearby. In the model of paints a similar effect can be proved using the same means. Therefore, we will only note that the manifestations of asymmetry between an inertial and non-inertial observers as a results of their observations. Formal use of formulas for calculation of simultaneous events and distances between them in two reference systems will result in the situation when the distance measured in the non-inertial reference system (ABC) will not change in the section \((B_1B_2)\). At the same time, the clock of the inertial observer in the section \((C_1C_2)\) will significantly gain in relation to the clock moving along the trajectory \((B_1BB_2)\). In other words, the non-inertial observer will consider that the clock of the “twin”, which is stationary relative to him, run much faster.
In the model of paints the similar effect means that the value of the mixture \((C_1C_2)\) is higher compared to the value of the sum of the two mixtures \((B_1B)\) and \((B_2B)\). It directly follows from Fig.6 on its own account. At the same time, the seeming paradoxicality is explained by the fact that the mixture \(C_1\) is equal in value to the mixture \(B_1\) relative to the scale of colour \((AB)\), while the mixture \(C_2\) is equal in value to the mixture \(B_2\) relative to the scale of colour \((BC)\). Thus, we cannot calculate the value of the mixture \((C_1C_2)\) if the values \(C1\) and \(C_2\) are measured according to different scales.

3.5. Value analog of motion in a homogeneous gravitational field

In physics the effect of slowing down of the clock rate in a gravitational field is opposite to the effect described in the “paradox of twins”. If, for instance, we compare the state of two “twins”, one of which remains of the surface of the Earth, and the other is in a flight-fall up and down in the gravity field of the Earth, the clock of the second twin will indicate a gain instead of slowing down! The most well-
known interpretation of this effect in connected with the curvature of the space-time in the gravity field. It eventually resulted in the development of the general theory of relativity, which postulates curvature of space in the vicinity of massive bodies.

Analysis of the analog of this effect in the theory of paints will allow us to become closer to the understanding of analogs of force fields and construction of dynamics on the basis of the variational approach. For simplicity, we will consider “motions” relative to the physical scale of the model of paints (equal volumes of “R” and “Y” paints have equal value). It means, for instance, that if we add to the mixture \( A_1 \) (Fig. 7) an equal quantity of “R” and “Y” paints, it will move to point \( A_2 \), change its colour, but will remain in the same point of the axis of quality \(<y>\) (the difference between the quantity of “R” and “Y” paints will remain unchanged). In other words, it will be conditionally (relative to the physical scale of value) “stationary”. If the prices in different points of the scale \(<y>\) are equivalent, then in case of changing of the composition of the mixture along the non-inertial (curved) trajectory, the “effect of twins” described above can be observed. At the same time, in case if the value of paints somehow depends on the scale \(<y>\) (the difference between the quantity of “R” and “Y” paints in the mixture), this dependency can be interpreted as a field of force.

Let us illustrate it by the following example. Let us first consider the simplest case of a linear dependency of the value of paints on the coordinate according to the axis of quality. Let, for instance, in case of displacement of the scale of value “A” for \( \Delta y \) (to the right in Fig.7), the physical volume of the units of measurement of value \( \delta V_0 = 1l \) decreases by the value of \( g \cdot \Delta y \).

\[
\delta V(y) = \delta V_0 - g \cdot \Delta y \tag{9}
\]

The value \( g \) in the model of paints is dimensionless, as the deflection of the mixture along the axis of “quality” \( \Delta y \) in the dedicated physical scale is measured in the same units of measurement as its value – in liters. Let us also note that in case of small displacements \( (\Delta y \to 0) \) any analytical dependency \( \delta V(y) \) can be approximated by the linear dependency (9). In this case, changing of \( \Delta S \) of the value of a certain mixture at the same changing of its physical volume \( \Delta V \) will increase:

\[
\Delta S(y) = \frac{\Delta V}{\delta V(y)} = \frac{\Delta V}{\delta V_0 - g \cdot \Delta y} \approx \frac{\Delta V}{\delta V_0} \left( 1 + g \cdot \frac{\Delta y}{\delta V_0} \right) \tag{10}
\]
At the same time, equal volumes of “R” and “Y” paints remain equally valuable. Therefore, the ratio of values of “R” and “Y” paints comprising the mixtures remain the same in case of displacement of $\Delta y$ and the colour of mixture relative to the displaced scale does not change. Such changing of the value scale range in the process of displacement to a different point can be interpreted as curvature of the initially flat space. In physics free bodies in a curved space move along geodesic lines (the shortest distance between points).

This requirement represents a particular case of the principle of least action. In the relativistic theory the action of a free material point can be written down as

$$\Delta S_{12} = -m_0 c \int_1^2 ds = -m_0 c^2 \int_1^2 dt_0$$  \hspace{1cm} (11)$$

where $m_0$ is the rest mass, $ds$ is the gain of the interval in the process of motion along the trajectory of motion, $t_0$ is the time measured by means of the associated clock,

Fig 7. Linear dependency of the price of paints on their qualitative composition results in the effect of a uniform potential force field and slowing down of the value-based clock in case of increase of its potential.
moving together with the material point. Thus, in case of inertial motion in a force field, the material point seemingly “selects” the trajectory, in which the associated clock will indicate the maximum time.

Proceeding to the theory of paints, we can apply the same variational principle for finding the optimum trajectory of motion. It provides a maximum gain of value according to the intrinsic (associated) scale of values. Therefore, it can be referred to as the principle of maximum benefit (PMB).

In the described example such trajectory represents a parabola. It corresponds to the uniformly accelerated motion. In case of small velocities in physics and “shades of colour” in the theory of paints we obtain a classical (non-relativistic) limit of the principle of least action and the principle of maximum benefit, respectively.

4. Example of a problem of classical dynamics, which can be solved in the model of paints

Let us consider one of the classical problems of dynamics on finding the trajectory of motion of a material point at the given initial and finite points of motion in the external field. The most general method of its solution is the variational method, in which the function of action of the system along the trajectory of its motion is minimized:

\[ \Delta S_{12} = \int_{1}^{2} L \, dt = \int_{1}^{2} \left( \frac{mv^2}{2} - U(y, t) \right) dt \] (12)

In this case, the potential energy, which can depend both on the coordinate and on time, is considered specified. In the example, discussed by us above, the similar problem of obtaining the maximum gain of value of a mixture is reduced to the expression (12) in case when the colour of the mixture \( v \) (analog of the velocity in (2)) does not differ significantly from the neutral colour (the quantity of «R» is almost equal to the quantity of «Y» and \( |v| \ll 1 \)), and the displacement of \( \Delta y \) is sufficiently small.

At the same time, in terms of the model of paints, for the small measurement of value of the mixture \( dS_{12} \) within a random section we can write down:
\[
dS_{12} = \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial y} dy = \frac{\gamma}{\delta v_0} dV + \frac{\delta V}{\delta v_0} \cdot g \frac{\Delta y}{\delta v_0} dy \approx \left(1 - v^2/2\right) + g \frac{\delta V}{\delta v_0} \Delta y dV \left(\frac{\delta V}{\delta v_0}\right) (13)
\]

We can assume that the value \(\frac{\delta V}{\delta v_0}\) in (13) acts as \(m dt\) in (12), while \(g \frac{\Delta y}{\delta v_0}\) – as \(\frac{U(y,t)}{m}\). At the same time, the value \(\delta V_0\) acts as the unit of measurement of value in the dedicated reference system. So far we do not consider in the model of paints the analog of physical mass and do not derive it according to \(\frac{\delta V}{\delta v_0} \leftrightarrow m dt\), as in this simple example it acts as a term in both summands of profit and does not influence the result of problem solution.

The practical realization of this solution can include a situation when a proprietor of a painting workshop sells surplus of paints of neutral colour. If in this case he always remains in the line A, then in the state A_2 he will receive profit \([A_2-A_1]\). However, in the states of the line B the price of the mixture he may be forced to sell due to some external reasons will be higher compared to the states of the line A (in accordance with (9)). Therefore, the proprietor, in order to increase his profit and for transition from A to B, in the section AB_1 (Fig. 7) sells only the paint “R”, then in the section B_1B_2 he receives increased profit from selling the neutral mixture, and, finally, in the section B_2A he sells only the paint «Y», in order to return to the initial state A.

In this interpretation the displacement of the endpoint of the vector of mixture state characterizes the changing of the surplus product (SP), obtained as a result of production and eventually sold. The payment received for its sale can be conditionally considered the received “benefit”. Then it becomes natural to call the variational principle of maximization of value gain of a mixture, measured according to a proper “clock”, the principle of maximum benefit (PMB).

Considering that in the inclined sections of the trajectory the proprietor receives smaller profit (or does not receive it at all as a result of slowing down to 0 of the associated “value-based clock”), the optimum value \(\Delta y\) exists, at which in point A_2 the profit turns out to be maximum. However, a rigorous solution of the variational problem allows receiving an even large profit (benefit), by moving along the “parabola” trajectory.
In general, the described situation looks unnatural. In the general case, the price of the paint can randomly depend both on the initial state of the proprietor (his coordinates of “R” and “Y” in the space of mixtures), and on the colour of the mixture obtained as the surplus product (analog of velocity set by the parameters «ΔR» и «ΔY»). An example of such random distribution of price in space is illustrated in Fig.3. However, even in this case we can introduce a corresponding analog of the force field (not necessarily potential), describe its properties of curved space and solve the problem of optimization using the variational principle of the maximum benefit.

In the present study we did not set the task of obtaining any new mathematical results. However, it has been shown by us that the two previously introduced postulates (*) and (**) are sufficient for considering the task of obtaining the maximum value as a variational problem of classical mechanics in the relativistic space of values. The mathematical apparatus required for solving such problems is described in detail, for instance, in [12].

**Conclusion**

In this study we have limited ourselves to the analysis of only geometric and kinematic effects associated with the representation of values as mixtures of two “pure” paints. It turned out that only two natural postulates on the absolute relation of values and on the absence of the dedicated colour are sufficient for constructing a relativistic space of states of such mixtures of the dimension (1+1). Even in such a simplest case we are able to solve classical variational problems of optimization by the analogy with problems of classical mechanics.

However, for a rigorous application of the physical methodology the theory of value is to be generalized and expanded. In our further studies we are planning to analyze the following trends of such generalizations:
C.1 Generalization for the multi-dimension space of values of (2+1) and (3+1) dimensionalities

In the general case the space of mixtures of pure paints forms a convex set of finite dimensionality. Pure paints are located on the boundary of this set. From the theory of convex analysis \cite{13, 14} it follows, in particular, that:

- For each mixture a corresponding set of representations in the form of mixture of only two pure paints (points of the boundary) exists.
- Any internal element of the set contains any element of the boundary.

It has been shown above, that as a result of synchronization of scales constructed for each pair of mixtures of pure paints, the product of values of two of its components remains invariant (analog of an interval in physics).

Transition to a scale constructed for a different pair of pure paints is equivalent to rotation of the “Einstein mirrors” in the physical space. The requirement of retaining the interval in case of such rotation allows proceeding to the analysis of the geometry of a multi-dimensional space of values (with dimensionality (2+1) and (3+1)). We can reasonably assume that in the view of the Frobenius theorem (real division algebras) the expansion (3+1) is maximally possible for preserving the previously introduced invariant of description (7).

C.2 Association of coordinates of states of a mixture in the space of value with physical time

In the process of construction of the relativistic space of values we have considered trajectories of “motion” as continuous sequences of states of mixtures connected by the absolute relation, without reference to real processes generating this sequence. However, changing of states in real processes is always connected with physical time. In particular, a transaction on exchange of values, as a basic element of interaction, can take place only simultaneously for both participants.

In this connection, in the relativistic space of values a dedicated reference system occurs, in which each state of the trajectory of “motion” is associated with a specific moment of physical time. In this dedicated system physically simultaneous states of
mixtures must be equal relative to the corresponding scale. The problem of coordination of physical simultaneousness and economic equivalence of states of economic objects will be analyzed in our further studies.

Besides, in each real transaction not only mixtures of equal values, but also those, the values of which differ absolutely - in a great number of times, can participate. In this case, not all of the mixture participate in the exchange, but only its certain part. After the exchange the obtained paint is, nevertheless, distributed through the whole volume of the mixture, and its price “velocity” is measured in inverse proportion to the quantity of the mixture. Thus, considering elementary transactions of exchange of a certain part of mixture for an equal part of a different mixture as collisions, we approach the understanding of price analogs of the dynamic variables: the impulse and the mass.

Application of the principle of maximum benefit to systems of interacting proprietors, for which the notions of price analogs of impulse, mass, energy, and others are defined, will allow writing down the variational principle of the maximum benefit in the form analogous to the classical equations of dynamics in the relativistic physical space. The preliminary results of such analysis have been previously published in [3, 4, 6, 15].

C.3 Quantum generalization of the procedure of measurement of value

Special attention should also be paid to the analysis of properties of the fundamental measurements. Previously, in [16, 17] we have constructed the algebra of transactions and have shown that a “composite transaction” (A+B), the conditions of which envisage consent for the transaction only in case if the proprietor agrees for (A) or for (B), is equivalent to the quantum-mechanical addition of their alternatives, rather than probabilities. In the opposite case, a riskless arbitration can be obtained in the result.

In particular, we have proposed an economic illustration of the well-known in quantum physics two-slit experiment, allowing experimental proving and corresponding measurement of the analog of the “de Broglie wavelength” of the observed object.
In the framework of the model of paints discussed above or its generalizations we expect to associate the “quantum-mechanical properties” of the subject of transaction with the properties of its “value clock”, namely – the effective distance between the “mirrors of the Einstein clock”. In general, the quantum-mechanical generalization of the laws of dynamics of motion in the relativistic space of economic states will allow proceeding to their representation in the operator form. One of such generalizations has been previously realized by us for the Black-Scholes-Merton formula in [18]. In that case we have used the formalism of the theory of continuous fuzzy quantum measurements [19].

References


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