Debt Risk And Pricing Of Risk

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Abstract:
The calculation of risk and its pricing has been of great interest to the financial field(s) of economics. As issuance of bonds, loans, mortgages etc. has accelerated so have schemes to bundle, diversify and hedge such 'assets' and with a similarly accelerated 'need' or ever more accurate calculation methods of the risk associated with default, correlation between assets, economic fluctuations of periods of downturns and its converse. With trillions of dollars in such assets outstanding, the 'need' for accuracy is understandable and perhaps mandatory.

Introduction:

Several risk valuations are popular and useful in certain limits...these range from portfolio optimization, valuation of derivatives and secondary and hedging and insurance instruments, and assets of debt and collateralized debt obligations.

There is criticism of all of these methods. It is somewhat justifiable, as they have all at one time or another failed 'spectacularly'. We take the 'enlightened' view that these approaches are sound in conceptualization yet are incomplete in methodology of mathematical description. That is, these are quite brilliant theoretical models, yet which fail when the limited mathematical descriptions are applied. These mathematical shortcomings stemming from the repeatedly faulty use of Gaussian statistics, and linear and logarithmic stochastics to model fluctuating variables entering into the various theories.

As complexity of human financial markets are subjected to uncertainties ranging from weather and storms and famine and plethora to wars and rumors of wars and supposedly less intensely to the celebrated beating of a Butterfly's wings across the Globe, the human financial enterprise is an open complex (n-Body, interacting nonlinear) and random (uncertainties, fluctuations) system and must for any reasonable accuracy incorporate the use of nonlinear stochastics, non-Gaussian statistics such as power-law and/or qExponential. As we have it, these mathematics have been derived and are well understood and are in use currently throughout the sciences and inclusively of finance.

We therefore state that the 'spectacular' failures of the financial economics models that have occurred in the past are the result of the limited capability of linearity (in stochastics) and exponential/logarithmic functions (Gaussian's, normal) in capturing outliers and the fat tailed statistics of outliers...Therefore many financial markets went ahead during times of quiescent fluctuations such as bull markets and expansions with the use of the financial models as published and were unable to quantify their risk properly when outliers events began to emerge, this resulting in their large scale failures, and as inter-dependent networks, in subsequent cascade effects such as the recent sub prime failure, the plunge of the stock markets and so on which occurred in recent memory 2007-2009.

Discussion:

We need then to

a) generalize existing giants' shoulders' theories to nonlinear and power law statistics which capture risk due to fat tails and outliers.

b) derive a sound and verifiable theory of debt.
c) derive a theory of correlated assets, given that there is a justified criticism of unobservable correlation between assets and therefore a real lack of an accurate risk pricing model of CDOs and debt instruments asset classes.

These a-c will be statistical dynamical as is the usual derivation of financial mathematics formula, for example the celebrated Black-Scholes derivatives pricing formula is a Legendre transform of stochastic variables yielding a (backwards) PDE partial differential F-P Fokker-Planck type equation.

Also these can be more detailed...for example correlation means interaction, means in detail an n-Body problem using physics jargon (n-assets, n-loans, n-investors, etc... with $0<n<N$ with N some large amount or number indicative or characteristic of system size such as market size, dollar amount of issued debt instruments and so forth).

Correlations meaning Interactions also means networks of interactions/correlations and as such we can use methods from network theory/models to describe these n-Body bundles or investors or the particular focus of derivation.

Discussion II:

What is the correlation between a bundle of loans say of mortgages...each loan is different in amount, is owned by different persons with different circumstances of age income job job security sex background demographics locale state urban rural etc..., have different maturity, are of different amounts...
a) critics state that financial valuation based on correlations (unobservable as they supposedly are) is tantamount to charlatanism.
b) yet many risk modeling by registered nationally recognized credit agencies rank and rate credit and loan instruments such as mortgage backed securities and CDOs utilizing formula that relies on mutual information that is correlations, often approximated and often replicated by comparison.
c) the Gaussian copula method and formula of tranches premiums and pricing of risk derived by X. (David) Li 2000 is yet a mainstay of such credit evaluations, although Li himself warned of its misuse and the general lack of understanding of the limitations of the theoretical model.
d) Many attempts have been made to generalize the Li theory with the purpose of obtaining a higher accuracy...as an aside, we also derive a generalization of the theory, pointing out the solution to the volatility clustering, volatility skew between downturns (when assets risk become much more highly correlated) and upturns (when correlations reduce) and tranche correlation risk...these presenting a problem summarized by the probability of defaults graph as a function of number of loans (or credit, Dietsch & Petey J. Banking Finance 26, 2002, 303-322) during three regimes of economic robustness

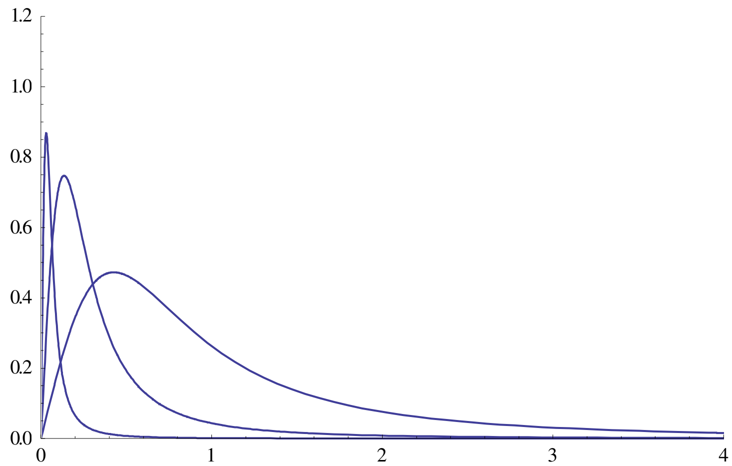

Fig.1. The probability k defaults (y-axis), and (x-axis) k(X100) defaults for three economies corresponding to the left-most 'sharp' lowest correlation 0.05, mid-level correlation 0.2 and the right-most correlation ~0.9.

that correspond (equivalently) to tranche correlation. Here we have the high correlation obtaining higher fat tailed probability distribution of defaults, for example correlation 0.9 has a ~20% probability of ~120 defaults while the mid correlation of ~0.2 has ~3-5% for the same ~120 defaults...over the time horizon of the issuances, here ~10years as in Duffie and Garleanu (Financial Analysts Journal, 2001).

Also we mention that tranche correlation is also related to credit rating and premium...The highest tranche has been found to have a low correlation initially as opposed to the lowest tranches, yet as reported (elsewhere) during times of economic downturns, the correlation of even the highest tranche increases to nearly the same level as the increases in the lowest tranches.

Discussion III:

The Li and Vasicek models, copula and dynamic models, are generalized by several methods...For the Copula model, of which a Loss function LF is derived dependent on the probability distribution of default PDD, the PDD is generalized by several authors to the Gamma distribution, to the Student-T distribution, and mixtures of such PDDs that attempt to capture the fat tails of default these dependent on a) correlation as discussed above dependant on economic 'climate', and time horizons and on tranches b) correlation 'smile' as a function of downturn-upturn...that is the correlation increases much more during downturns than it does during upturns.

These generalizations noted and as are the motivations for generalizing the one factor

Gaussian copula model, we mention that the Gamma and Student-T distributions bear a very close relationship to the nonextensive and superstatistics, these dynamically robust statistics in that the SDEs stochastic diffeqs

$$dx(t) = a(x,t)dt + P(x,t)^{(\frac{1-q}{2})} dW(t)$$

are i) nonlinear stochastic ii) exhibit the nonlinear feedback between macro PDF statistics P(x,t) and the stochastics Ito sense dx(t) iii) are F-P Fokker-Planck nonlinear evolved as

$$\nabla_t P(x,t) = -\nabla_x a(x,t) P(x,t) + \frac{1}{2} \nabla_x^2 P(x,t)^{2-q}$$

and v) are PDF distributed as a fat tail distribution of the power-law type (here a(x,t)=a.x)

$$P(x,t) = \frac{1}{Z_q} [1 + \beta(t)(q-1)(x - c.t)^2]^{(\frac{1}{1-q})}$$

.

The copula method is obtained as by the P(y)dy=P(x)dx transformation from uniform distribution to here nonextensive and its inverse...we have the uniformly distributed to nonextensive distribute random numbers

$$\text{y(x)= x Hypergeometric2F1}[\frac{1}{2}, \frac{1}{q-1}, \frac{3}{2}, \beta(t)(1-q)(x^2)]$$

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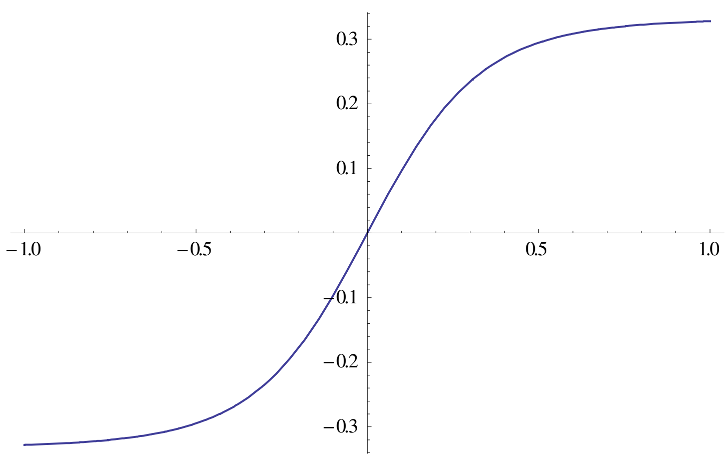


which generally is the shape of the copula functions as they are CDFs of the distributions, for example the Cauchy or Gaussian or Gamma and so forth all derive the smooth to sharp 'hyperbolic' like functions that when sharply parameterized tend towards a Heaviside step function (the derivative of which is a Dirac Delta function).

The one factor Gaussian copula model can then be generalized straightforwardly as in the several extended models such as the Gamma and Student-t.

We note and importantly for our purposes and towards credit risk in CDSs and CDOs valuations...

a) The SDE stochastic equation of the Ito sense is nonlinear...

b) it has an equivalent macro statistics description of the q-nonlinear PDE Fokker-Planck like 'Tsallis-Zanette' equation.

c) with linear mean for illustrative purposes where a(x,t) generally nonlinear and time dependent itself, is however made linear a(x,t)=c.x, the SDE and PDF are yet q-nonlinear, with q-parameter 'tuning' the degree of nonlinearity, heavy-tails, superdiffusion, nonlinear macro statistics to stochastics trajectory history following feedback and other viewpoints.

d) the cumulative CDF distribution function is sharp as q-parameter, with higher q corresponding to a sharper hyperbolic (tanh(...)) like behavior approaching a step or Heaviside function...and as q->1 obtaining the normal or Gaussian CDF.

e) with a(x,t)==0 both the SDE and equivalently the PDF is yet nonlinear...this means that if with symmetry (as with a(x,t)=c.x yet symmetrical about the mean) being preserved, a differing q'>q value will yet produce the same fit of power-law distribution for the data. That is:

$$dx(t) = a(x,t)dt + P(x,t)^{(\frac{1-q}{2})} dW(t)$$

$$dx'(t) = P(x',t)^{(\frac{1-q'}{2})} dW'(t)$$

are 'equivalent' SDEs along different trajectories (will be set equal), the second SDE of q'>q simpler to evaluate ofcourse as it is a Fokker-Planck of the form

$$\nabla_t P_{q'}(x,t) = \frac{1}{2}\nabla_x^2 P_{q'}(x,t)^{2-q'}$$

which is solved by

$$P_{q'}(x,t) = \frac{1}{Z_{q'}}[1+\beta(t)(q'-1)(x^2)]^{(\frac{1}{1-q'})}$$

This transformation is made by

Alternatively Ito formula transformation to a de-meaned SDE as by Ito formula

$$dx'(x) = \nabla_x x' dx + \frac{1}{2}\nabla^2{}_x x' dx^2$$

$$dx' = [\nabla_x x' a(x,t) + \frac{1}{2}\nabla^2{}_x x' P(x,t)^{(1-q)}]dt + \nabla_x x' P(x,t)^{(\frac{1-q}{2})} dW(t)$$

and with another change of variables by the Ito formula

$$dh = 0 + 1/2\nabla^2{}_{x'} h dx'^2 = 1/2\nabla^2{}_x h B^2(x,t)$$

$$B^2(x,t) = \frac{1}{4a^2(x,t)}[P(x,t)^{(3-3q)}]$$

$$P_q(x,t) = [4a^2(x,t)P_{q'}(x,t)^{1-q'}]^{\frac{1}{(3-3q)}}.$$

These transformations while useful are merely for illustration...the nonlinearity of heavy tails due to superdiffusion of assets say is incorporated in the zero mean q' power law nonextensive distribution for a univariate and moreover for multivariates. Ad hoc, we merely utilize the zero mean q' fully nonlinear and tunable (heavy tails) distribution...its advantages are immediately obvious, a superdiffusion statistics/stochastics and information/entropy theory that is exactly solvable and which reduces to the normal Gaussian statistics in the q'->1 limit.

Discussion VI.b:

We wish to apply these statistics to credit default swaps and collateralized debt obligations. These we seek to perform as we report elsewhere these power-law statistics of nonextensivity and its two parameter variant the superstatistics are well adapted to superdiffusion observed in assets equities and markets, and which in the q-parameterized Tsallis statistics case are fully information theoretic making it possible to identify this with incomplete information as befitting real world markets and other human systems. Also as we have shown elsewhere these nonextensive statistics allow us to derive a fully 'accurate' Random Matrix Theory which allows us to interpolate between Poisson, Gaussian and chaotic heavy tailed regimes this a recent innovation to the Wigner and Wishart random matrices which we introduced in 2001 and 2002. This is fortunate as real debt default distributions are reported by several authors as realistically from real corporate and markets data is distributed as 'running the gamut' from sharply peaked near zero Poisson low correlations to mid level Gaussian like for middle correlations and to heavy tailed and low probability of default near zero for high correlations, these giving effect to several features in real debt and assets markets such as correlation skew, tranche vulnerability as a function of correlation, and economic defaults distribution skew/divergence during economic downturns or recessions or crashes and times of economic upturns.

However note immediately that junior and mezzanine tranches are nearly immune to these effects as the heavy tails at outliers and yet near zero probabilities of defaults near low defaults due to the high correlation say qRMT nonextensive distribution of defaults we presented above has in its central regions a Gaussian like (though lower in magnitude) regime and as is reported Gaussian defaults do fit regular upturn periods well enough, and which leaves the junior and mezzanine tranches with a near identical (lower) risk exposure as for Gaussian and power-law nonextensive/super/random matrix.

However the behavior for equity is markedly different...The low probability of default in low loans or assets levels makes it apparent that Gaussian evaluation of risk for equity tranches demand much higher rates of risk payments and cost compared to the nonextensive statistics evaluation.

Derivation VII: nonextensive copula model.

The copula model of loss and default is simple to write down yet has shown limitations (as reported elsewhere) due mainly to the nonexistent capability of Gaussian's in modeling outliers risk, and therefore extreme events such as crashes, recessions, downturns, bankruptcies etc...

A copula is formed from the CDF cumulative distribution function of a univariate or multivariate distribution, as transformed from an independent or uncorrelated statistics to a correlated, nonlinear and heavy tailed statistics, and its inverse transformation.

We note that a copula of uniformly distributed statistics is written as

$$C(u_1,...,u_i,...,u_n;R)=\prod_{i=1}^{n}u_i \quad .$$

The univariate density function PDF is obtained from a CDF cumulative distribution as

$$f(x_i)=\frac{\partial F(x_i)}{\partial x_i}$$

$$F(x_i)=\int_0^{x_i} f(x'_i)dx'_i \quad .$$

Sklar has shown that inverses are possible between transformations of copulas as we discussed, and that for continuous variables the copula relationship to its univariate and joint correlated distributions and CDF is unique.

We have for a derivation of the nonextensive copula the following:

$$f_q(x_i) = \frac{1}{Z_q}[1+\beta(t)(q-1)(x_i^{\,2})]^{(\frac{1}{1-q})}$$

$$F_q(x_i) = x_i \; Hypergeometric2F1[\frac{1}{2}, \frac{1}{q-1}, \frac{3}{2}, \beta(t)(1-q)(x_i^{\,2})]$$

$$f_q(x_1,...,x_i,...,x_n) = \frac{1}{Z_q}[1+(q-1)(\vec{x}.\vec{R}.\vec{x}\,')]^{\left(\frac{1}{1-q}\right)}$$

.

The superscripts are dropped in favor of q, the 'P' is exchanged for 'f' to follow customary notation, the CDF 'F' is the hyperbolic (tanh(..)) -like Hypergeometric tunable by q, q itself obtained from real markets' data, and the multivariate distribution is written compactly in vector notation and where the 'R' matrix is the correlation matrix, simply enough corresponding to the variate's co-dependence as by the diffusion coefficients.

$$dx_i(t) = f_q(\vec{x},t)^{(\frac{1-q}{2})} dW(t)$$

$$dx_j(t) = f_q(\vec{x},t)^{(\frac{1-q}{2})} dW(t)$$

$$\vec{x} = x_1\hat{e}_1 + ... + x_i\hat{e}_i + ... x_1\hat{e}_n$$

$$i, j \le n$$

and where to explicitly show the 'source' of correlation in stochastics-to-statistics, consider the bivariates nonlinear SDEs

$$dx(t) = b(x, y, t)dW(t)$$
$$dy(t) = c(x, y, t)dW'(t)$$

$$\nabla_t P(\vec{x}, t \mid \vec{x}', t') = \frac{1}{2}\vec{\nabla}^2_{\vec{x}} B^2{}_{\vec{x}} P(\vec{x}, t \mid \vec{x}', t')$$
$$\nabla_{t'} P(\vec{x}, t \mid \vec{x}', t') = \frac{1}{2} B^2{}_{\vec{x}'} \vec{\nabla}^2_{\vec{x}'} P(\vec{x}, t \mid \vec{x}', t')$$
$$X'(x', y', t') = \int_0^x \frac{1}{B^2{}_{\vec{x}'}} dx''$$
$$\nabla_{t'} P(\vec{X}, t \mid \vec{X}', t') = \frac{1}{2}\vec{\nabla}^2_{\vec{X}'} P$$
$$P(\vec{X}, t \mid \vec{X}', t') = \frac{e^{-\left(\frac{(X-X')^2+(Y-Y')^2}{2(t-t')}\right)}}{\left(\sqrt{4\pi(t-t')}\right)^2}$$
$$\vec{x}.\vec{R}.\vec{x} = (X - X')^2 + (Y - Y')^2$$

which obtain one form of the diffusion coefficient as a coupling (read in stochastics as diffusion coefficients (conditional moments' variances and co-variances) and in statistics of copulas as 'correlation') between bivariates and more generally univariates.

We see again then the 'beauty' of the q-parameterized nonlinearity...The diffusion coefficients are in actuality the PDF itself as discussed, making i) exact solutions possible ii) simple correlation structures 'immediately' obtainable iii) high accuracy with real data and the quantifying of outliers simplified.

We therefore have the derivation continued as the nonextensive copula

$$c(x_1, ..., x_n) = \frac{f_q(x_1, ..., x_n)}{\prod_{i=1}^{n} f_q(x_i)}$$
$$\frac{\partial^n [C(F(x_1)..\partial F(x_n)]}{\partial F(x_1)..\partial F(x_n)} = c(x_1, ..., x_n)$$
$$c(x_1, ..., x_n) = \frac{\frac{1}{Z_q}[1 + (q-1)(\vec{x}.\vec{R}.\vec{x}')]^{\left(\frac{1}{1-q}\right)}}{\prod_{i=1}^{n} \frac{1}{Z_q}[1 + (q-1)(x_i)^2]^{\left(\frac{1}{1-q}\right)}}.$$

with the normalization as the inverse of the partition function and going as Gamma functions

$$Z_q(t) = \frac{B(\frac{1}{2}, \frac{1}{q-1} - \frac{1}{2})}{\sqrt{(q-1)\beta(t)}}$$

$$\beta(t) = \frac{1}{2\sigma_q(t)^2 Z_q(t)^{q-1}}$$

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

,

and the ordinary variance and the q-variance entering into these expressions are averaged over different measures yet of the nonextensive statistics, this discussed in our references.

Discussion VI:
Additionally the dynamic latency of company stochastic dynamic model of Vasicek can also be generalized, in fact to this copula model, as the stochastic model is of the loan default, and if one utilizes the nonextensive (nonlinear information/entropy composition) SDE above, its PDD of loan default will be the nonextensive power-law distribution.

Discussion V:
The loan default correlation is criticized as unobservable...however on the average as in any uncertain dynamics, its value can be modeled, extracted from data and its mean and variance ascertained. This in fact constitutes the statistical dynamical modeling discussed above.

However these statistics, summarized by observables such as <L(t)> the average value of the i'th CDO or loan in a basket or bundle at time t must follow the observed heavy tailed distributions of real data of defaults as discussed, again the motivation for extending the /Gaussian copula to student-t, Gamma, inverse Gaussian and in our letter the nonextensive power-law statistics.

Discussion VIII:
The statistical dynamics discussed, we move to the interacting and network pictures that produce the observables multivariates which we derived in the sections above.

The model we will derive is simply the mean field magnetization model, in discrete form the Ising model, however generalized to discrete+continuous interactions and whereas the fluctuations uncertainties are nonextensive. We have previously reported such a model for interacting n-Investors and n-opinion or n-information of financial markets, and we will apply it to credit and instruments of credit in this letter.

We define the default of a loan as its 0 state, and its survival as 1 and which we rewrite symmetrically as -1(default) and +1(survival) this discrete model can be generalized to continuous variables readily and we discuss this later.

An ensemble of loans will then have an average survival or default due to all of its individual loans, which must be averaged as an observable as <M(t)>==survival ('magnetization in physics') and which has the further definition of

$$< m(t) >= \frac{< N_u(t) > - < N_d(t) >}{< N(t) >}$$

$$< m(t) >= \frac{< M(t) >}{< N(t) >}$$

$$< N(t) >=< N_u(t) > + < N_d(t) >$$

and where <..> denotes averaging probabilistically.

The interactions between these undefaulted=u and defaulted=d loans, amounting to their correlations as discussed in the above sections, is nonlinear as

$$V = -\sum_{i,j\neq i=1}^{N} k_{ij}\sigma_i\sigma_j$$

$$V_o = -k_o \sum_{i=1}^{N} \sigma_i N < \sigma >$$

$$< m(t) >=< \sigma(t) >$$

which we simplify by linearizing the interaction to the 'mean field' model whereas the interaction correlation is also simplified as a common strength this corresponding to a uniform correlation between bivariates in copula Gaussian one factor models, however in this derivation the nonuniformity or nonlinearity will enter as by q-parameterization, while the interactions are linearized such that each loan is correlated with the entire loan asset class, or basket, or market, this data often easily obtained by several means from CDX credit swaps indices to Moody's loan default (time dependent) data. We also define m(t) the survivability per loan instrument whereas the M(t) is the entire collection of loans as defined and m=M/N.

The usual method in continuous models at this point is to write the minimum Shannon information measure constrained by the interaction, and as negative entropy is information, or the maximization of entropy as

$$< I(t) >= -c.\sum_{i=1}^{N}\int P(x,t)\ln P(x,t)$$

$$< I_q(t) >= -c.\sum_{i=1}^{N}\frac{\int (f-f^q)}{(1-q)}$$

$$\delta < I(t) > +\delta[\beta\{(x-<x>)^2+\alpha<V_o>\}] == 0$$

$$P_{\sigma_i}(x_i,t) = \frac{e^{\left(-\frac{(x_i-<x_i>)^2-k_oNm(t)\sigma_i}{2D(t-t_o)}\right)}}{\sqrt{4\pi D(t-t_o)}}$$

$$\beta(t) = \frac{1}{2D(t-t_o)}$$

$$< m(t) >=< \sigma(t) >= \int P_u(x,t)dx - \int P_d(x,t)dx$$

$$< m(t) >= Tanh[\beta k_o Nm(t)]$$

which is the well known result in linearized or equivalently mean-field magnetization models and obtains hyperbolic (tanh(...)) as should be expected for symmetrical survival/default. Here additionally it is weighted as by the inverse temperature which in stochastics (by Einstein and Wick) is the inverse variance which can be derive from the diffusion coefficient.

The q parameter nonextensive case is a bit more algebra, however not much is different as would be expected for a nonlinear formalism which in the q->1 limit must obtain this Gaussian result. The q-entropy is maximized as before and the PDF obtained is

$$f_{\sigma_i}(x_i,t)_q = \frac{1}{Z_q(t)}[1+\beta(t)(q-1)\{(x_i-<x_i>)^2-k_oNm(t)\sigma_i\}]^{\frac{1}{(1-q)}}$$

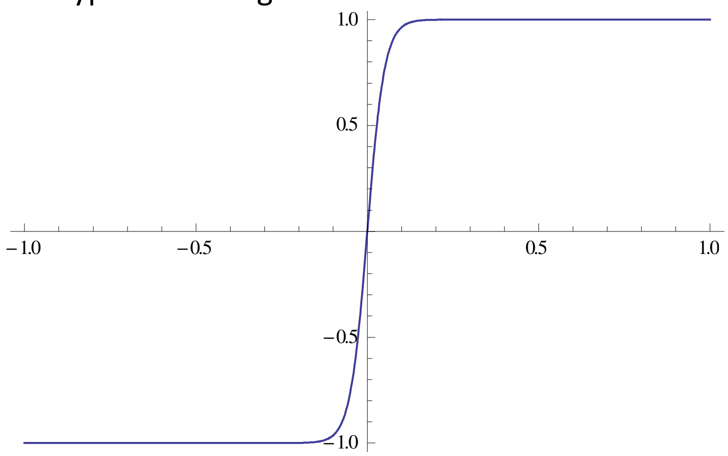
which for <m(t)> is with a change of variables

$$< m(t) >= \frac{\sqrt{\pi}(1+\beta(t)\sigma k_o Nm(t)(q-1))^{\frac{1}{1-q}}\Gamma[-\frac{-3+q}{2(-1+q)}]}{\sqrt{\frac{\beta(t)(q-1)}{1-\beta(t)\sigma k_o Nm(t)(q-1)}}\Gamma[\frac{1}{-1+q}]}$$

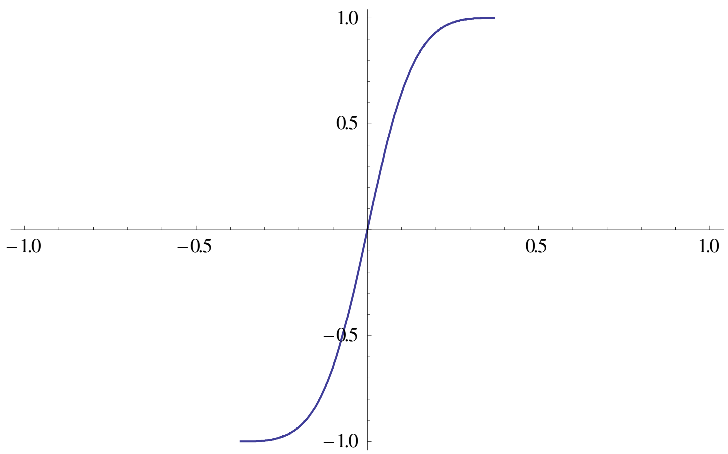
This when expanded produces the hyperbolic tanh(..) like of CDF like behavior expected.

A modification here is that the m(t) is self referring and generally numerical root finders must be utilized.

The hyperbolic tangent as from Gaussian mean field is



while for the same variance the outlier and tails of the power law distribution obtain



Conclusion IX:

This obtains a variance (qvariance) of the survival of a loan in time until its maturity. There is a similar approach which initially makes each N(t), Nu(t) and Nd(t) time dependent stochastic, and that is referenced.

With a probability of loan value and survival probability continuous here but discrete jumps can be straightforwardly written down, either by discrete to continuous Fourier transform or by discrete probabilistic (master equation) approaches.

The remaining derivation is then to utilize the probability of survival/default in a Loss function calculation per tranche or asset class or group...This is as usual written as a risk function as

The points to be made here are:

a) Any risk can be quantified with knowledge of the Loss function and probability of loss as

$$\Upsilon(t) = \int LdP$$

$$\approx \int L(x_1,..,x_n)P(x_1,..,x_n)dx_1,..,dx_n$$

...the probability of loss or probability of survival is obtained by several methods in this paper. Specifically we discuss the generalization of Gaussian and Student-t copulas to nonextensive statistics these well adapted to heavy tails quantification of outlier extreme events risk. We also mention that this is similar to recent results in the mixed factor Student-t copula approaches, and can be viewed as a robust and theoretically founded approach. As additionally it allows for Random Matrix probability of default and as from nonextensive and superstatistical approaches.

We additionally have discussed briefly a micro detailed interactions model of Loans instruments...these are linearized such that simply each loan is correlated with the entire averaged environment of default and risk, pointing however towards future generalization in as much that detailed nonlinear interactions can be partitioned as discussed in the statistical dynamics case between drift and diffusion coefficients of the random processes or here between potentials of interaction between loans (deterministic's) and fluctuations (stochasctics and statistics).

Also we mention that future research will take us into network theory...this by the way obtains in the mean field structure node/links models as we have recently shown the Gaussian mean field, and beyond it the power-law networks of nodes of investors and or loan instruments and or credit and the links or structure of interactions probabilistic or deterministic that occur between their 'trees'. These networks whether as abstract as information inter nets and social can be made 'geometric' where hamming distances between information nodes measure as metrics the distance. This can be graphed, This has a geometrical metric interpretation and therefore we can obtain Lagrangian and Hamiltonian that is Louisvillian operator formalisms that are the 'hidden laws' of such metrics of networks. These are identically then the interacting deterministic+stochastic models we are discussing, and a Fokker-Planck equation for the nonextensive statistics copula, the 'magnetic' like loans or inter actors models, and the hidden metrics of topology of information networks obtain the same results in the mean fields or in the nonlinear regimes. These as discussed produce skews, heavy tails, and additional or reduced risk depending on which structured asset level an investment in the case of swaps straightforwardly risk valuation is made as a function of such skews and un symmetric dynamics, while in the case of credit obligations collateralized these are dependent on the way the structure of the investment is made such that tranches experience different levels of risk and moreover experience it differently than the

Gaussian 'tail-less' models

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